7-1 Defining and evaluating logarithms

- I can convert between exponential and logarithmic form

- I can evaluate logarithms

$\overset{y_1}{5000} = \overset{y_2}{1000}(1+.05)^t$

logs = exponents

Explain 1 Converting Between Exponential and Logarithmic Forms of Equations

In general, the exponential function $f(x) = b^x$, where $b > 0$ and $b \neq 1$, has the logarithmic function $f^{-1}(x) = \log_b x$ as its inverse. For instance, if $f(x) = 3^x$, then $f^{-1}(x) = \log_3 x$, and if $f(x) = \left(\frac{1}{4}\right)^x$, then $f^{-1}(x) = \log_{\frac{1}{4}} x$. The inverse relationship between exponential functions and logarithmic functions also means that you can write any exponential equation as a logarithmic equation and any logarithmic equation as an exponential equation.

Exponential Equation	**Logarithmic Equation**
exponent	answer
$b^x = a$	$\log_b a = x$
base, answer	base, exponent

$$b > 0,\ b \neq 1$$

"log base b of a equals x"

Examples

Exponential Equation	Logarithmic Equation
$4^3 = 64$	$\log_4 64 = 3$
$5^{-2} = \frac{1}{25}$	$\log_5 \frac{1}{25} = -2$
$\left(\frac{2}{3}\right)^p = q$	$\log_{\frac{2}{3}} q = p$
$\left(\frac{1}{2}\right)^n = m$	$\log_{\frac{1}{2}} m = n$

Switch between Log and exponential forms

Exponential Equation	Logarithmic Equation
$3^5 = 243$	$\log_3 243 = 5$
$4^{-3} = \frac{1}{64}$	$\log_4 \frac{1}{64} = -3$ exp
$\left(\frac{3}{4}\right)^r = s$	$\log_{\frac{3}{4}} s = r$
$\frac{1}{5}^w = v$	$\log_{\frac{1}{5}} v = w$ exp

The natural logarithm:

$$y = \ln_e x \quad \text{is equivalent to} \quad x = e^y$$

exponent

Base e

The common logarithm:

exponent

$$y = \log_{10} x \quad \text{is equivalent to} \quad x = 10^y$$

Base 10

Exponential Equation	Logarithmic Equation
$e^5 \approx 148.4$	$\ln 148.4 \approx 5$
$e^{1.8} \approx 6$	$\ln 6 \approx 1.8$
$10^5 = 100{,}000$	$\log 100{,}000 = 5$
$10^3 = 1{,}000$	$\log 1{,}000 = 3$

Evaluating logarithms

$f(x) = \log x$ Find f(10), f(0.1), f(100)

$f(10) = \log_{10} 10 = ?$

$10^{?} = 10$

$? = 1$

$f(0.1) = \log 0.1 = ?$

$10^{?} = \frac{1}{10}$ OR .1

$10^{-1} = \frac{1}{10}$ $? = -1$

$f(100) = \log 100 = ?$

$10^{?} = 100$

$? = 2$

Evaluate f(x)

$f(x) = \log_2 x$ Find f(4), f(16), f(64)

$\log_2 4 = \cancel{?}\,2 \qquad 2^? = 4 \qquad ? = 2$

$\log_2 16 = \cancel{?}\,4 \qquad 2^? = 16 \qquad ? = 4$

$\log_2 64 = \overset{6}{\cancel{?}} \qquad 2^? = 64$

Evaluate f(x)

$f(x) = \log_7 x$ Find f(49), f(343)

$\log_7 49 = \cancel{?}$ 2

$7^{?} = 49$

$\log_7 343 = \cancel{?}$ 3

$7^{\cancel{?}\,3} = 343$

Find the exact value

$\log_2 32 = x$

$2^x = 32$

$x = 5$

$\log_4 \frac{1}{16} = x$

$4^x = \frac{1}{16}$

$x = -2$

$\log 10{,}000{,}000 = x$

$10^x = 10{,}000{,}000$

$x = 7$

$\log .00001 = x$

$10^x = .00001$

$x = -5$

$10^1 = 10$

$10^0 = 1$

$10^{-1} = .1$

$10^{-2} = .01$

Find the exact value

$\log_5 25 = 2$

$5^2 = 25$

$\log_2 \frac{1}{8} = -3$

$2^{-3} = \frac{1}{8}$

$\log 1000 = 3$

$10^3 = 1000$

$\log .001 = -3$

$10^{-3} = .001$

Use a calculator to

First, find the common logarithm of 0.42. Round the result to the thousandths place and raise 10 to that number to confirm that the power is close to 0.42.

$\log 0.42 \approx$ -.376

$10^{\square} \approx 0.42$

Next, find the natural logarithm of 0.42. Round the result to the thousandths place and raise e to that number to confirm that the power is close to 0.42.

$\ln 0.42 \approx$ -.867

$e^{-.867} \approx 0.42$

Your Turn

Use a scientific calculator to find the common logarithm and the natural logarithm of the given number. Verify each result by evaluating the appropriate exponential expression.

11. 0.25

$\log .25 = -.602$

$10^{-.602} \approx .250$

$\ln .25 = -1.386$

$e^{-1.386} \approx .2500\ldots$

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