

8-5 Inverse Functions

Objectives:

- I can find the inverse of a given function
- I can verify that an inverse is a function

Inverse of a Relation

The **inverse of a relation** consisting of the ordered pairs (x, y) is the set of all ordered pairs (y, x) .

Notation:

$$f^{-1}(x) =$$

$$g(x)$$

$$g^{-1}(x)$$

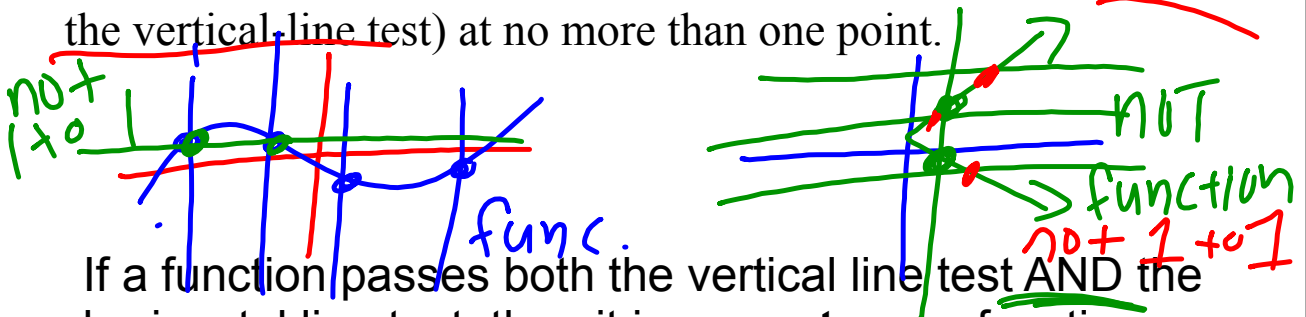
Represents the inverse of the function

$$f(x)$$

+ , -

Horizontal-Line Test

The inverse of a function is a function if and only if every horizontal line intersects the graph of the given function (passed the vertical-line test) at no more than one point.

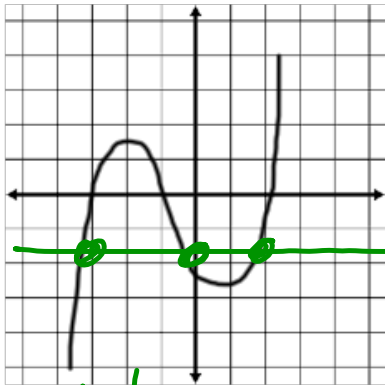


If a function passes both the vertical line test AND the horizontal line test, then it is a **one-to-one** function.

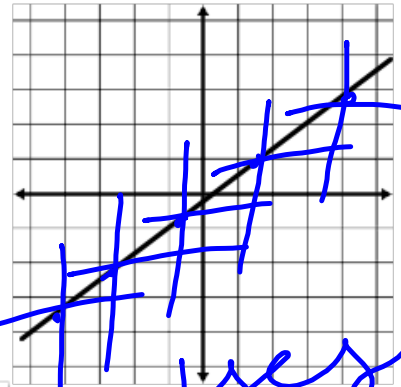
$f(x)$
function

$f^{-1}(x)$
function

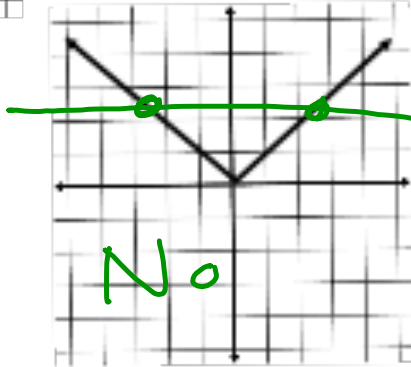
Determine whether the function is one-to-one.



No



yes!



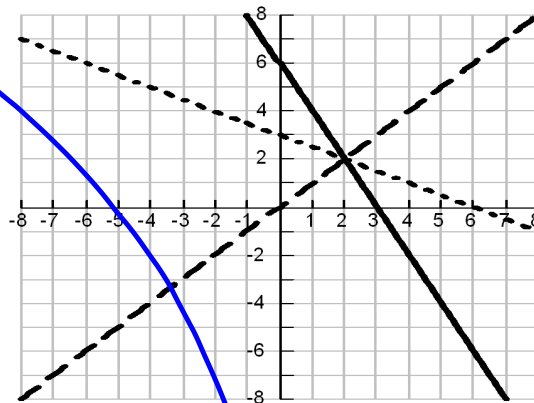
No



Inverses - graphically

Inverse relations are reflections of each other over the line $y = x$ (identity function)

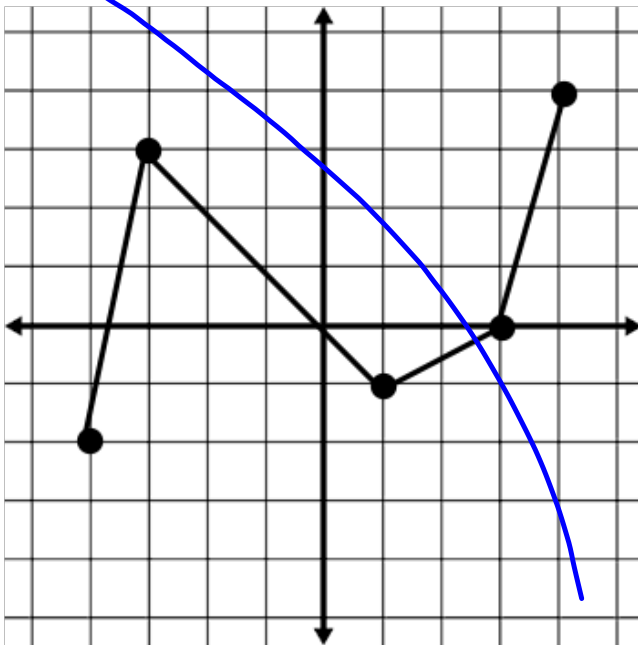
Show $f(x) = 6 - 2x$ and $g(x) = \frac{6-x}{2}$ are inverses graphically



$f(x):$	$(1, 4)$	$(3, 0)$	$(4, -2)$
$g(x):$	$(4, 1)$	$(0, 3)$	$(-2, 4)$

X X X

Graph the inverse of the graph. (Use $y=x$ to find inverse points)



To find the inverse equation of a function

1. Change $f(x)$ to y .
2. Interchange x and y
3. Solve for y - get y by itself
4. Change new y to $f^{-1}(x)$

Given $a^n = b$

If n is odd: $a = \sqrt[n]{b}$

If n is even: $a = \pm \sqrt[n]{b}$

Find the inverse of each function

$$f(x) = x^2 + 1$$

$$y = x^2 + 1$$

$$x = y^2 + 1$$

$$\sqrt{x-1} = y$$

$$\sqrt{x-1} = y$$

$$\sqrt{x-1} = f^{-1}(x)$$

$$f^{-1}(x) = \sqrt{x-1}$$

~~$$g(x) = 2x - 3$$~~

$$y = 2x - 3$$

$$x = 2y - 3$$

$$\frac{x+3}{2} = y$$

$$\frac{x+3}{2} = g^{-1}(x)$$

Find the inverse of each function.
Is the inverse a function?

$$h(x) = 2x^3 + 3$$

$$y = 2x^3 + 3$$

$$x = 2y^3 + 3$$

$$\frac{x-3}{2} = \frac{2y^3}{2}$$

$$\sqrt[3]{\frac{x-3}{2}} = y$$

$$\sqrt[3]{\frac{x-3}{2}} = h^{-1}(x)$$

$$g(x) = \sqrt[3]{x} - 3$$

$$y = \sqrt[3]{x} - 3$$

$$x = \sqrt[3]{y} - 3$$

$$(x+3)^3 = y^3$$

$$(x+3)^3 = g^{-1}(x)$$

Find the inverse of each function.
Is the inverse a function?

$$\cancel{f(x) = 2x + 5}$$

$$\cancel{g(x) = \sqrt{x-7}}$$

$$y = \sqrt{x-7}$$

$$x^2 = \sqrt{y-7}$$

$$x^2 = y - 7$$

$$x^2 + 7 = y$$

$$x^2 + 7 = g^{-1}(x)$$

Find the inverse of each function.
Is the inverse a function?

$$f(x) = \sqrt{2x - 3}$$

$$g(x) = 3x^2 + 2$$

Find the inverse of each function.
Is the inverse a function?

$$f(x) = (x + 3)^2$$

$$f(x) = (x - 5)^3$$

