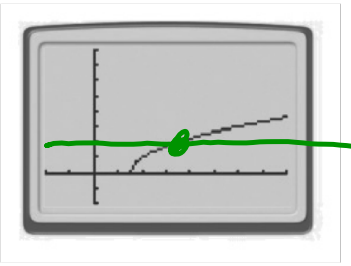
8-4 Solving Radical Equations

Objectives:

1. I can solve radical equations and ~~check for extraneous solutions~~.

Remember that you can graph the two sides of an equation as separate functions to find solutions of the equation: a solution is any x-value where the two graphs intersect.

The graph of $y = \sqrt{x - 3}$ is shown on a calculator window of $-4 \leq x \leq 16$ and $-2 \leq y \leq 8$. Reproduce the graph on your calculator. Then add the graph of $y = 2$.



How many solutions does the equation $\sqrt{x - 3} = 2$ have? _____ How do you know?

On your calculator, replace the graph of $y = 2$ with the graph of $y = -1$.

How many solutions does the equation $\sqrt{x - 3} = -1$ have? _____ How do you know?

Graph both sides of $\sqrt{4x-4} = x+1$ as separate functions on your calculator.

How many solutions does $\sqrt{4x-4} = x+1$ have? _____

Replace the graph of $y = x+1$ with the graph of $y = \frac{1}{2}x$.

How many solutions does $\sqrt{4x-4} = \frac{1}{2}x$ have? _____

Replace the graph of $y = \frac{1}{2}x$ with the graph of $y = 2x-5$.

How many solutions does $\sqrt{4x-4} = 2x-5$ have? _____

Solving Analytically

Solve.

$4\sqrt{x} - 6 = 6$

$4\sqrt{x} - 6 + 6 = 6 + 6$

$4\sqrt{x} = 12$ Simplify.

Now divide each side by 4 to isolate the radical.

$4\sqrt{x} = 12$

$\sqrt{x} = 3$ Divide and simplify.

Next, square each side of the equation to eliminate the radical.

$(\sqrt{x})^2 = 3^2$

$x = 9$ Simplify.

Finally, check $x = 9$ in the original equation to verify that it is a solution and not an extraneous solution.

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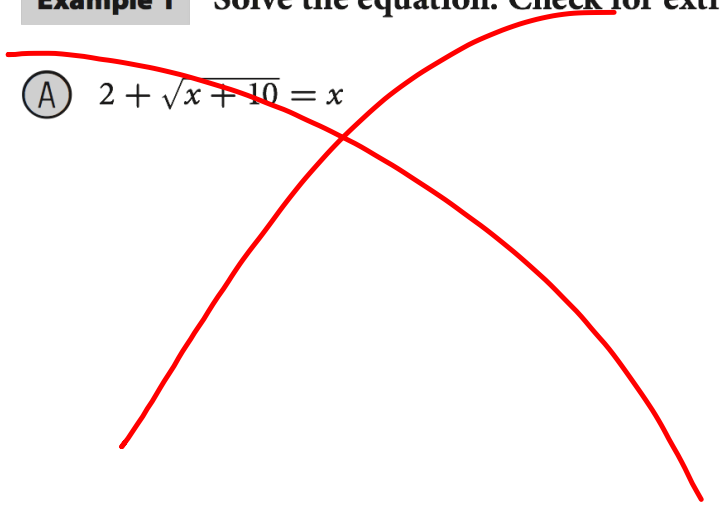
Example 1 Solve the equation. Check for extraneous solutions.

(A) $2 + \sqrt{x + 10} = x$



B $(x+6)^{\frac{1}{2}} - (2x-4)^{\frac{1}{2}} = 0$

$\sqrt{x+6} - \sqrt{2x-4} = 0$

$+\sqrt{2x-4} \quad +\sqrt{2x-4}$

$(\sqrt{x+6})^2 = (\sqrt{2x-4})^2$

$x+6 = 2x-4$

6. Solve $(x+5)^{\frac{1}{2}} - 2 = 1$.

$\sqrt{x+5} - 2 = 1$

$+2 \quad +2$

$(\sqrt{x+5})^2 = 3^2$

$x+5 = 9$

$-5 \quad -5$

$x = 4$

Solve the following, check for extraneous solutions

$2\sqrt{x} = 3\sqrt{x-2}$

$\sqrt{5x-11} = x-1$

$\sqrt{2x+5}+4=3$

$-4 \quad -4$

$\left(\sqrt{2x+5}\right)^2 = -1^2$

$2x+5 = 1$

$-5 \quad -5$

$\frac{2x}{2} = \frac{-4}{2}$

$x = -2$

Example 2 Solve the equation.

(A) $\sqrt[3]{x+2}+7=5$

$-7 \quad -7$

$\sqrt[3]{x+2} = -2^3$

$x+2 = -8$

$-2 \quad -2$

$x = -10$

B $\sqrt[3]{x-5} = x+1$

Your Turn

8. Solve $2(x-50)^{\frac{1}{3}} = -10$.

$\frac{2\sqrt[3]{x-50}}{2} = \frac{-10}{2}$

$\sqrt[3]{x-50} = -5$

$(\sqrt[3]{x-50})^3 = (-5)^3$

$x-50 = -125$

$+50 \quad +50$

$x = -75$

$(2\sqrt[3]{x-50})^3 = -10^3$

$8(x-50) = -1000$

Solve the following:

$$\sqrt[3]{x-5} = \sqrt[3]{7-x}$$

$x - 5 = 7 - x$

$2x = 12$

$x = 6$

$$\sqrt[3]{x+2} = \sqrt[3]{x+3}$$

$x + 2 = x + 3$

No Solution

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