8-3 Exponential Review

I can apply exponential properties and use them

I can model real-world situations using exponential functions

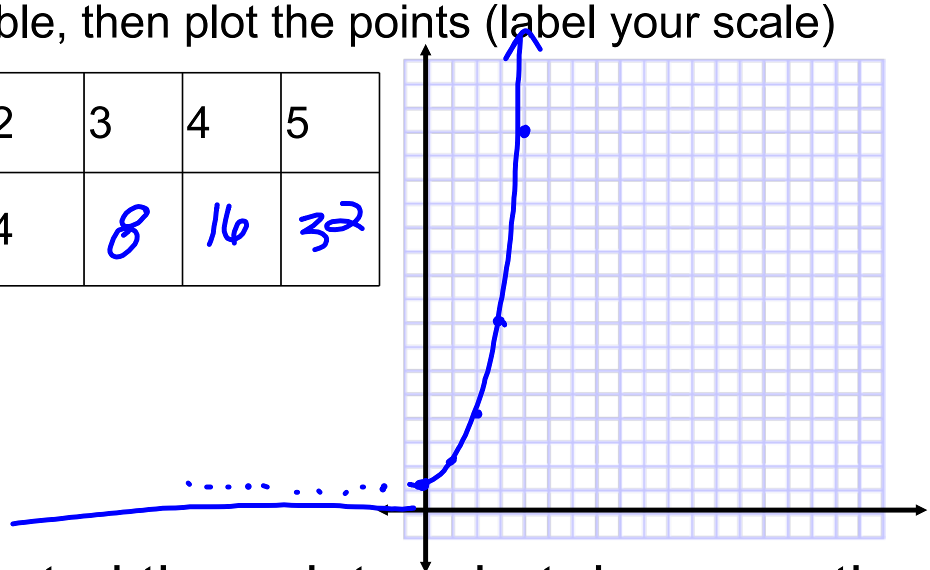
Warm-Up

1. Find the next three terms in the sequence

   2, 6, 18, 54, 162, 486, 1458

2. Fill in the table, then plot the points (label your scale)

n	0	1	2	3	4	5
f(n)	1	2	4	8	16	32



If we connected the points, what do you notice about the graph? exponential

Have you ever seen a graph like this before?

yes!

EXPONENTIAL FUNCTION

$$f(x) = a(b)^{x}$$

Exponent

Initial Value
(y-intercept)

Base
(Multiplier)

Graph the following functions on a calculator and sketch.
Be sure to plot the y-intercept

a. $f(x) = 1 \cdot 2^x$

b. $f(x) = 1 \cdot \left(\frac{1}{2}\right)^x$

growth

decay

What did you notice about the graphs and their equations?

$\frac{7}{4}$

Exponential Growth and Decay

$f(x) = a(b)^x$

When $b>1$, the function represents **exponential growth**

When $0<b<1$, the function represents **exponential decay**

Determine whether each function represents growth or decay

a. $f(x) = 13\left(\frac{1}{3}\right)^x$

decay

b. $g(x) = \left(\frac{3}{2}\right)^x$

growth

Write one equation that represents growth and one that represent decay

growth

$f(x) = \left(\frac{4}{2}\right)^x$

decay

$f(x) = 2\left(\frac{1}{4}\right)^x$

general

Exponential Growth/Decay Equation

$$f(t) = a\underbrace{(1 \pm r)}_{b}^{t}$$

f(t): final amount

a: initial amount

r: Rate AS A DECIMAL

t: time

$\pm$: growth (+) decay (-)

John researches a baseball card and find that it is currently worth \$3.25. However, it is supposed to increase in value 11% per year.

$$f(t) = a(1 \pm r)^t$$

a) Write an exponential equation to represent this situation

$$f(t) = 3.25(1+.11)^t$$

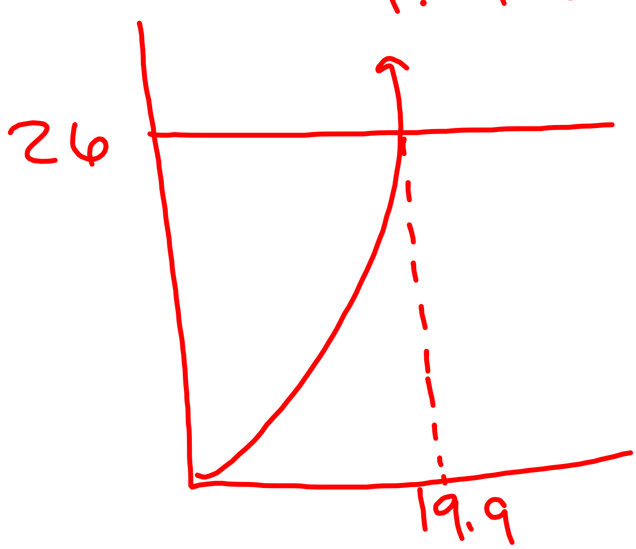
b) How much will the card be worth in 10 years?

$$f(10) = 3.25(1.11)^{10} = \$9.25$$

c) Use your graphing calculator to determine in how many years will the card be worth \$26.

$$26 = 3.25(1.11)^t$$

19.93 yrs



You Try!

On federal income tax returns, self employed people can depreciate the value of business equipment. Suppose a computer valued at $2765 depreciates at a rate of 30% per year.

$$f(t) = a(1 \pm r)^t$$

a) Write an exponential equation to model this situation

.7

$f(t) = 2765(1 - .3)^t$

b) How much will this computer be worth in 5 years?

$f(5) = 2765(.7)^5 = \$464.71$

c) Use your graphing calculator to determine in how many years will the computer be worth $350.

5.79 yrs

The population of Orem in 1950 was 4,000 and was increasing at a rate of 2.6% per year.

a) Predict the population of Orem in 1975 and 2000.

b) Using your graphing calculator, predict when Orem's population will hit 200,000 people.

The half-life of Carbon-14 is 5700 years. If a fossil decayed from 15 grams to 1.875 grams, how old is the fossil? (use your calculator)

$$f(t) = a(1-r)^t$$

$$\underset{y_1}{1.875} = 15\underset{y_2}{(1-.5)^t}$$

t = # of half lives

3 half lives

$3 \times 5700 = 17,100$ yrs

Compound Interest Formula

$$A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$$

P is the principal

r is the annual interest rate as a decimal!

n is the number of compounding periods per year

t is the time in years

annually $n = 1$
semi-annually $n = 2$
monthly $n = 12$
quarterly $n = 4$
weekly $n = 52$
daily $n = 365$
bi-annually $n = \frac{1}{2}$

Write an equation then find the final amount for each investment.

a. $1000 at 8% compounded semiannually for 15 years

$$A(t) = 1000\left(1 + \frac{.08}{2}\right)^{2 \cdot 15}$$

$$A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$$

$3,243.40

You Try!

b. $1750 at 3.65% compounded daily for 10 years

365

$$A(t) = 1750\left(1 + \frac{.0365}{365}\right)^{365 \cdot 10}$$

$ = 2520.85

c. Using a calculator, determine how many years it will take for the amount to reach $4000.

A(t)

$$4000 = 1750\left(1 + \frac{.0365}{365}\right)^{365 \cdot X}$$

y_1 y_2

22.6 yrs

Investigate the growth of $1 investment that earns 100% annual interest (r=1) over 1 year as the number of compounding periods, n, increases.

Compounding schedule	n	$1\left(1+\frac{1}{n}\right)^n$	Value of A
annually	1		
semiannually	2		
quarterly	4		
monthly	12		
daily	365		
hourly	8760		
every minute	525600		

What does the value of A approach?

The value e is called the natural base

The exponential function with base e, $f(x)=e^x$, is called the natural exponential function.

$$e \approx 2.71828182827$$

what you need to know is $e \approx 2.7$

Evaluate $f(x)=e^x$ for

a. $x = 2$ 7.389

b. $x = \frac{1}{2}$

c. $x = -1$

Many banks compound the interest on accounts daily or monthly. However, some banks compound interest continuously, or at every instant, by using the *continuous compounding formula.*

Continuous Compounding Formula

If P dollars are invested at an interest rate r, that is compounded continuously, then the amount, A, of the investment at time t is given by

$$A(t) = Pe^{rt}$$

P = initial amount

r = rate as decimal

t = time

A person invests $1550 in an account that earns 4% ~~annual~~ interest compounded continuously.

a. Write an equation to represent this situation

$$A(t)=1550e^{.04\cdot t}$$

b. Using a calculator, find when the value of the investment reaches $2000.

$$A(t)=Pe^{rt}$$

$$2000=1550e^{.04t}$$

An investment of $1000 earns an annual interest rate of 7.6%.

Compare the final amounts after 8 years for interest *compounded quarterly* and for interest *compounded continuously.*