8-2 Finite Geometric Series

w/ end multiply

Objectives:

1. I can write a series with sigma notation.

2. I can derive the formula for the sum of a geometric series (when the common ratio is not 1)

3. I can use the formula of a geometric series to solve problems.

Warm - up

1. Write a recursive rule and an explicit rule for the sequence:

9, 27, 81, 243

$\cdot 3 \quad \cdot 3 \quad \cdot 3$

$a = 3 \quad r = 3$

$e: f(n) = 3 \cdot 3^n$

$R: f(n) = f(n-1) \cdot 3$

$f(0) = 3$

2. Find the stated term of the geometric sequence:

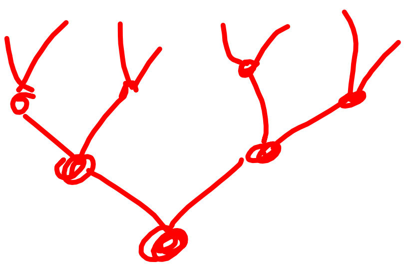
-3, -6, -12, -24, … ; 9th term

$\cdot 2 \quad \cdot 2$

$f(9) = -3/2 \cdot 2^9 = -768$

You have 2 biological parents, 4 biological grandparents, and 8 biological great-grandparents. How many great-great-great-great grandparents (6th generation) do you have?

sequence



0	1	2	3	4	5	6
1	2	4	8	16	32	64

How many direct ancestors do you have if you trace your ancestry back 6 generations?

$$\underline{2+4+8+16+32+64=126}$$

SUM

Paper Task

Start with a rectangular sheet of paper and assume the sheet has an area of 1 square unit. Cut the sheet in half and lay down one of the half-pieces. Then cut the remaining piece in half, and lay down one of the quarter-pieces as if rebuilding the original sheet of paper. Continue the process: At each stage, cut the remaining piece in half, and lay down one of the two pieces as if rebuilding the original sheet of paper.

Stage	Sum of the areas of the pieces that have been laid down	Difference of 1 and the area of the remaining piece
1	$\frac{1}{2}$	$1 - \frac{1}{2} = \square$
2	$\frac{1}{2} + \square = \square$	$1 - \square = \square$
3	$\frac{1}{2} + \square + \square = \square$	$1 - \square = \square$
4	$\frac{1}{2} + \square + \square + \square = \square$	$1 - \square = \square$

Follow - Up

1. Write the sequence formed by the areas of the individual pieces that are laid down. What type of sequence is it?

2. In the table from Step B, you wrote four related finite geometric series

$$\frac{1}{2}, \frac{1}{2}+\frac{1}{4}, \frac{1}{2}+\frac{1}{4}+\frac{1}{8}, \text{and } \frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}$$

One way you found the sum of each series was simply to add up the terms. Describe another way you found the sum of each series.

3. If the process of cutting the remaining piece of paper and laying down one of the two pieces is continued, you obtain the finite geometric series:

$$\frac{1}{2}+\frac{1}{4}+\dots+\left(\frac{1}{2}\right)^n$$

at the *nth* stage. Use your answer to the previous question to find the sum of this series.

Series:

def: sum of the terms in a sequence

sum: usually a total of a finite number of items added together

Summation

$$a_1 + a_2 + a_3 + \ldots + a_n$$

(how do we write the sum of long lists of numbers?)

$\sum$ sigma means summation

LAST TERM #

FIRST TERM

LAST TERM

Summation notation: $\displaystyle\sum_{k=1}^{n} a_k = a_1 + a_2 + a_3 + \ldots + a_n$

1ST TERM #

explicit Rule

Vocabulary

Upper bound

Sigma

$$\sum_{k}^{n} a_k$$

Index

Argument

Read as:

The sum of a_k from k to n

Find the following sums:

a. $\sum_{k=1}^{5} 3k = 3(1)+3(2)+3(3)+3(4)+3(5)$

$3 + 6 + 9 + 12 + 15 = 45$

1 2 3 4 5

b. $\sum_{k=5}^{8} k^2 = 5^2 + 6^2 + 7^2 + 8^2$

$25 + 36 + 49 + 64 = 174$

5 6 7 8

c. 2+5+8+11+...+29

1 2 3 4 10

14 + 17 + 20 + 23 + 26

$29 = \boxed{-1+3n}$

$30 = 3n$

$n = 10$

$\sum_{n=1}^{10} -1+3n$

Formula for Finite Geometric Series

$$\sum_{k=1}^{n} a_k = a_1 + a_2 + a_3 + \ldots + a_n$$

$$= \frac{a_1\left(1 - r^n\right)}{1 - r}$$

(teacher discretion for proof)

$$\sum_{k=1}^{n} a_k = a_1 + a_2 + a_3 + \ldots + a_n$$

$$= \frac{a_1\left(1 - r^n\right)}{1 - r}$$

Find the sum of the series:

$5+15+45+135+405+1215$

$$\sum_{n=1}^{6}(5/3)(3^n)$$

$$\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots+\frac{1}{512}$$

8

$\frac{1}{32}+\frac{1}{64}+\frac{1}{128}+\frac{1}{256}$

$$\sum_{n=1}^{8}\left(\frac{1}{2}\right)\left(\frac{1}{2}^n\right)=$$

$$\frac{\frac{1}{512}}{\frac{1}{2}}=\frac{\cancel{\frac{1}{2}}}{\frac{1}{2}}\cdot\left(\frac{1}{2}\right)^n$$

$$\frac{1}{256}=\frac{1^n}{2^n}$$

$n=8$

Your turn: $1-2+4-8+16-32$

$$\frac{1}{2}-\frac{1}{4}+\frac{1}{8}+\ldots-\frac{1}{256}$$

Niobe is saving for a down payment on a new car, which she intends to buy a year from now. At the end of each month, she deposits \$200 from her paycheck into a dedicated savings account, which earns 3% annual interest that is applied to the account balance each month. After making 12 deposits, how much money will Niobe have in her savings account?

Sum of a Finite Arithmetic Sequence:

$$\sum_{k=1}^{n} a_k = a_1 + a_2 + a_3 + \ldots + a_n$$

$$= \frac{n(a_1 + a_n)}{2}$$

Find the sum of the arithmetic sequence

-8, -1, 6, 13, 20, 27,...

117, 110, 103, ...,33

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