8-2: Properties of Logarithms

I can understand the properties of logarithms and use them to simplify logs.

I can apply multiple properties to a single logarithm.

$$b^x = a \rightarrow \log_b a = x$$

Find the value of each logarithm without using a calculator.

1. $\log_7 7 \rightarrow 7^x = 7 \quad x = 1$

2. $\log_{18} 18 \rightarrow 18^x = 18 \quad x = 1$

3. $\log_5 1 \rightarrow 5^x = 1 \quad x = 0$

4. $\log_9 1 \quad 9^x = 1 \quad x = 0$

log of 1 = 0   log = base

$$\log_a 1 = 0 \qquad \log_a a = 1$$

Evaluate

$\log_5 1 = 0$

$5^x = 1$

$\ln 1 = 0$

$e^x = 1$

$\log_4 4 = 1$

$\log_{10} 10 = 1$

Evaluate the logarithm:

1.2

1. $\log_3 3^2 = 2$

$3^x = 3^2$

1.8

2. $\log_5 5^8 = 8$

$5^x = 5^8$

Without evaluating, predict what the following logs equal:

3. $\log_2 2^{10} = 10$

$2^x = 2^{10}$

4. $\log_{20} 20^7 = 7$

$20^x = 20^7$

Inverse Property of Logarithms
If b and r are positive real numbers, with $b \neq 0$, then

$$\log_a a^r = r$$

Evaluate

$$\log_4 4^3 = 3$$

$$4^x = 4^3$$

$$\ln_e e^{-0.5} = -0.5$$

$$e^x = e^{-.5}$$

How would we write the following exponential as a log?

$$\overset{b}{\cancel{5^{\log_5}}}\, 20 \rightarrow\ = a$$

$b^x = a \qquad \log_b a = x$

$\log_5 a = \log_5 20$

$a = 20$

Inverse Property of Logarithms
If b and M are positive real numbers, with $b \neq 0$, then

$\cancel{\log_{20} 20^7}$

$$b^{\log_b \cancel{M}\, a} = \cancel{M}\, a$$

Evaluate

$$5^{\log_5 20} = 20$$

$$\cancel{8^{\log_8}} \sqrt{23} = \sqrt{23}$$

$$\cancel{12^{\log_{12}}} \sqrt{2} = \sqrt{2}$$

$$\cancel{10^{\log}} 0.2 = 0.2$$

Exponent Rules Review

$2^5 \cdot 2^3 = 2^8$ ADD exponents

$\frac{2^5}{2^3} = 2^2$ SUBTRACT " "

$(2^3)^5 = 2^{15}$ Multiply " "

$\sqrt[3]{8} = 8^{\frac{1}{3}}$ $\sqrt[n]{a} = a^{\frac{1}{n}}$

Product Rule of Logarithms
If M, N and b are positive real numbers, with $b \neq 0$, then

$$\log_b(M \cdot N) = \log_b M + \log_b N$$

Which exponent rule is this similar to?

Sum of exponents $a^b \cdot a^c = a^{b+c}$

Why would we want to be able to split up a logarithm?

$$\log_4(6x) =$$

$$\log_4(6) + \log_4 x$$

To Solve.

Write each of the following logarithms as the sum of logarithms.

$\log_2(5 \cdot 3)$

$= \log_2(5) + \log_2(3)$

$\ln(6z)$

$\ln(6) + \ln(z)$

$\log_4(9 \cdot 5)$

$\log_4(9) + \log_4(5)$

$\log(5w)$

$\log_{10}(5) + \log_{10}(w)$

How do you predict we would write the following logarithm as two logarithms?

$$\log_b \frac{M}{N} = \log_b M - \log_b N$$

Quotient Rule of Logarithms
If M, N and b are positive real numbers, with $b \neq 0$, then

$$\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$$

$\log_2\left(\frac{5}{3}\right) = \log_2 5 - \log_2 3$

$\log\left(\frac{y}{5}\right)$ $\log(y) - \log(5)$

$\log_7\left(\frac{9}{5}\right)$ $\log_7 9 - \log_7 5$

$\ln\left(\frac{p}{3}\right)$ $\ln(p) - \ln(3$

Write the following as the sum or difference of logarithms.

$$\log_3\left(\frac{4x}{y}\right) = (\log_3 4 + \log_3 x) - \log_3 y$$

$$\log_3\left(\frac{3m}{n}\right) = (\log_3 3 + \log_3 m) - \log_3 n$$

$$\log_3\left(\frac{q}{3p}\right) = \log_3 q - (\log_3 3 + \log_3 p)$$

Show that the following logs are equal:

Math ↑ ↑ enteR

$$\log_2 (4)^3 = 3 \cdot \log_2 4$$

$$\log_2(4^3) = 3 \cdot \log_2 4$$

$$\log_2 64 = 3 \cdot \log_2 4$$

$$2^{?} = 64 \quad = \quad 3 \cdot \cancel{2^{2} = 4}$$

$$x = 6 \quad = \quad 6$$

Power Rule of Logarithms
If M and b are positive real numbers, with $b \neq 0$, then

$$\log_b M^r = r \cdot \log_b M$$

Use the power Rule of Logarithms to express all powers as factors.

$5 \cdot \log_8 3^5 = 5 \cdot \log_8 3$

$\ln x^{\sqrt{3}} = \sqrt{3} \cdot \ln x$

$\log_2 5^{1.6}$

$1.6 \cdot \log_2 5$

$\log b^5$

$5 \cdot \log b$

Expand the logarithm.

$\log_2(x^2 \cdot y^3)$

$\log_2 x^2 + \log_2 y^3$

$2\log_2 x + 3\log_2 y$

$\log\left(\frac{100x}{\sqrt{y}}\right)$ $\sqrt{y} = y^{\frac{1}{2}}$

$(\log 100 + \log x) - \frac{1}{2}\log y$

$\log_4(a^2 b)$

$2 \cdot \log_4 a + \log_4 b$

$\log_3\left(\frac{9m^4}{\sqrt[3]{n}}\right)$

$(\log_3 9 + 4\log_3 m) - \frac{1}{3}\log_3 n$

Write each of the following as a single logarithm.

$\log_6 3 + \log_6 12 = \log_6 (3 \cdot 12)$

$\log(x-2) - \log x = \log\left(\frac{x-2}{x}\right)$

$\log_5 x - 3\log_5 2 = \log_5\left(\frac{x}{2^3}\right)$

$\left(\log(x-1) + \log(x+1)\right) - 3\log x$

$\log\left(\frac{(x-1)(x+1)}{x^3}\right)$

Rewrite and express in terms of a and b given that a=ln3 and b=ln4

ln12

ln16

How do we evaluate logs in a calculator??

Change of Base Formula

If $a \neq 0$, $b \neq 0$, and M are positive real numbers, then

$$\xcancel{\log_a M = \frac{\log_b M}{\log_b a}}$$

which means: *

$$\log_a M = \frac{\log M}{\log a} = \frac{\ln M}{\ln a}$$

Use your calculator to approximate the following:

$\log_4 45 = \dfrac{\log 45}{\log 4}$

$\log_3 75 = \dfrac{\log 75}{\log 3}$

$\log_6 40 = \dfrac{\log 40}{\log 6}$

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