

8-1 Radicals

- I can simplify radical expressions

8-1 Radicals

Definition
 n th root

$$\sqrt[n]{b} = a \text{ means } b = a^n$$

- if $n \geq 2$ and even then a and b must be greater than or equal to 0. (positive)
- if $n > 3$ and odd, then a and b can be any real number.

In $\sqrt[n]{b}$:

The symbol $\sqrt{\quad}$ is called the radical

n is called the index - groups of n

b is called the radicand

if there is no written index, an index of 2 is implied

Know your powers and roots

Perfect Squares:

$$1^2 = 1$$

$$2^2 = 4$$

$$3^2 = 9$$

$$4^2 = 16$$

$$5^2 = 25$$

Square Roots:

$$\sqrt{1} = 1$$

$$\sqrt{4} = 2$$

$$\sqrt{9} = 3$$

$$\sqrt{16} = 4$$

$$\sqrt{25} = 5$$

Perfect Cubes:

$$1^3 = 1$$

$$2^3 = 8$$

$$3^3 = 27$$

$$4^3 = 64$$

$$5^3 = 125$$

Cube Roots:

$$\sqrt[3]{1} = 1$$

$$\sqrt[3]{8} = 2$$

$$\sqrt[3]{27} = 3$$

$$\sqrt[3]{64} = 4$$

$$\sqrt[3]{125} = 5$$

Evaluate

$$\sqrt[2]{9} = 3$$

A handwritten prime factorization of 9. The number 9 is written in black. Two red lines branch from the 9 to the number 3, which is written twice. The two 3s are circled in red.

$$\sqrt[4]{16} = 2$$

A handwritten prime factorization of 16. The number 16 is written in black. A red line branches from the 16 to the number 4, which is written twice. From each 4, two red lines branch to the number 2, which is written four times. The four 2s are circled in red.

$$\sqrt[3]{64} = 4$$

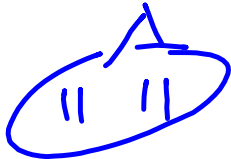
A handwritten prime factorization of 64. The number 64 is written in black. A red line branches from the 64 to the number 8, which is written twice. From each 8, three red lines branch to the number 2, which is written six times. The entire factorization is circled in blue.

$$\sqrt[3]{-8} = -2$$

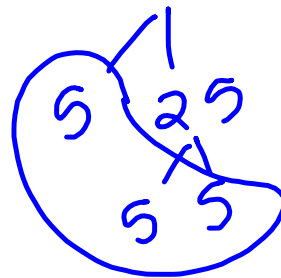
A handwritten prime factorization of -8. The number -8 is written in black. A blue line branches from the -8 to the number 4, which is written once, and the number 2, which is written once. From each of these, two blue lines branch to the number 2, which is written four times. The entire factorization is circled in blue.

You try

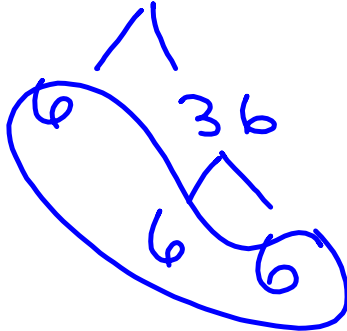
$$\sqrt{121} = 11$$



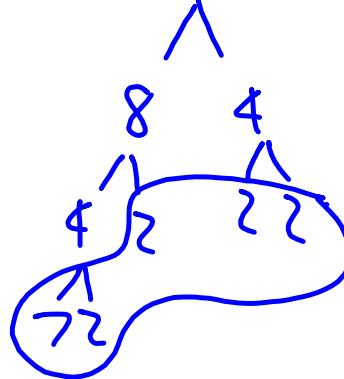
$$\sqrt[3]{125} = 5$$



$$\sqrt[3]{-216} = -6$$

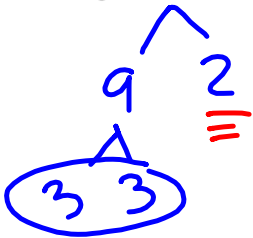


$$\sqrt[5]{32} = 2$$

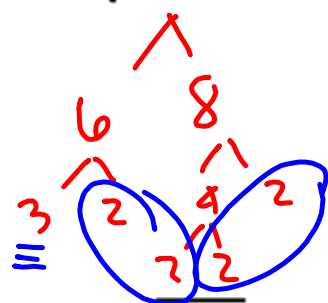


Simplify

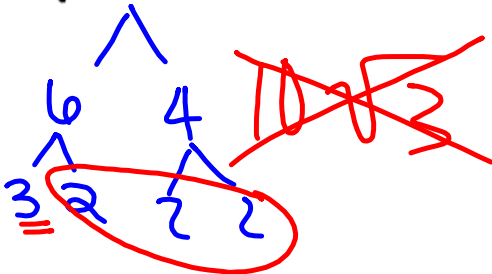
$$\sqrt{18} = 3\sqrt{2}$$



$$\sqrt{48} = 4\sqrt{3}$$



$$2. \quad 5\sqrt[3]{24} = 10\sqrt[3]{3}$$



$$\sqrt[4]{32} = 2\sqrt[4]{2}$$



Simplifying

If $n \geq 2$ is a positive integer and a is a real number, then

$$\sqrt[n]{a^n} = a \quad \text{if } n \geq 3 \text{ is odd}$$

$$\sqrt[n]{a^n} = |a| \quad \text{if } n \geq 2 \text{ is even}$$

For example

$$\sqrt{x^2} = |x| \quad \sqrt[3]{x^3} = x \quad \sqrt[4]{x^4} = |x| \quad \text{and so on}$$

*

But to make our life easier some instructions will say "Assume all variables are greater than or equal to zero." In which case:

$$\sqrt{x^2} = x \quad \sqrt[3]{x^3} = x \quad \sqrt[4]{x^4} = x \quad \text{and so on}$$

SO READ YOUR INSTRUCTIONS!!!

Reduce. Assume all variables are greater than or equal to zero.

$$\sqrt{x^2} = x$$

$$\sqrt[5]{x^5} = x$$

$$\sqrt[3]{x^3} = x$$

$$\sqrt[6]{z^6} = z$$

Reduce Assuming all variables are greater than or equal to zero.

(You can either do these using rational exponents or not.)

$$\sqrt[2]{x^6} = x^3$$

$$\sqrt[3]{x^{12}} = x^4$$

$$\sqrt[3]{x^{10}} = x^3 \sqrt[3]{x}$$

3R1

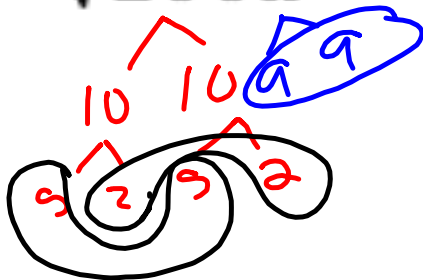
$$\sqrt[4]{x^{14}} = x^3 \sqrt[4]{x^2}$$

3R2

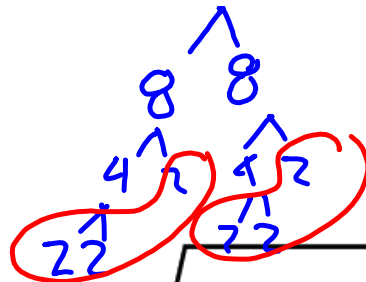
You try

(remember $\sqrt{x^2} = |x|$)

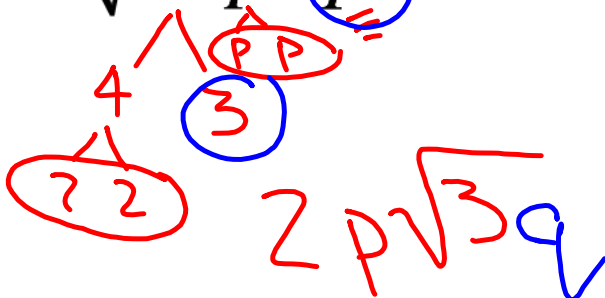
$$\sqrt{100a^2} = 10a$$



$$\sqrt[3]{64x^3} = 4x$$

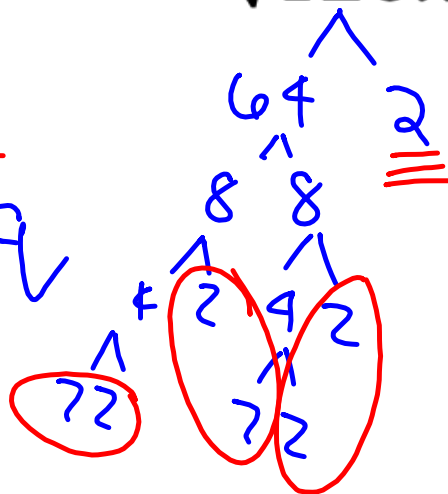


$$\sqrt{12p^2q}$$



$$2p\sqrt{3q}$$

$$\sqrt{128x^2}$$



$$8x\sqrt{2}$$

Reduce Assuming all variables are greater than or equal to zero.

$$\sqrt{20x^{10}} = 2x^5\sqrt{5}$$

Handwritten work for the first equation: The number 20 is broken down into 4 and 5. The number 10 is broken down into 2 and 8. The number 4 is circled, and the number 2 is written below it. The number 8 is written below the 2, and the number 2 is written below the 8. The number 2 is circled, and the number 2 is written below it.

$$\sqrt{75a^6} = 5a^3\sqrt{3}$$

Handwritten work for the second equation: The number 75 is broken down into 25 and 3. The number 6 is broken down into 2 and 4. The number 25 is circled, and the number 5 is written below it. The number 4 is written below the 2, and the number 3 is written below the 4. The number 3 is circled, and the number 3 is written below it.

Simplify Assuming all variables are greater than or equal to zero.

$$\sqrt{80a^3} = 4a\sqrt{5a} \quad \sqrt[3]{27m^4n^{14}}$$

$\sqrt{80a^3}$ is simplified to $4a\sqrt{5a}$. The prime factorization of 80 is $2^4 \cdot 5$. The square root of 2^4 is $2^2 = 4$. The square root of 5 is $\sqrt{5}$. The square root of a^3 is $a \sqrt{a}$.

$$\sqrt[3]{27m^4n^{14}} = 3m\sqrt[3]{n^2}$$

$\sqrt[3]{27m^4n^{14}}$ is simplified to $3m\sqrt[3]{n^2}$. The cube root of 27 is 3. The cube root of m^4 is m . The cube root of n^{14} is $n^4 \sqrt[3]{n^2}$.

$$\sqrt[5]{128x^6y^{10}} \quad \sqrt[4]{16a^5b^{11}}$$

$\sqrt[5]{128x^6y^{10}}$ and $\sqrt[4]{16a^5b^{11}}$ are not fully simplified in the image.