7-2 Properties of Logarithms

I can use the properties of logarithms to simplify logarithms.

I can use the properties of logarithms to express logarithms in different ways.

logs = exponents

$b^x = a$ $\log_b a = x$

$x^2 \cdot x^5 = x^7$

$\frac{x^6}{x^2} = x^4$

$x^0 = 1$

7-2 Properties of Logarithms

$$\log_a 1 = 0 \qquad \log_a a = 1$$

$a^0 = 1$ $\qquad$ $a^1 = a$

Log of 1 = 0 $\qquad$ Log of itself = 1

Evaluate

$\log_5 1 = 0$ $\qquad$ $\ln 1 = 0$

$5^x = 1$ $\qquad$ $e^x = 1$

$\log_4 4 = 1$ $\qquad$ $\log 10 = 1$

$4^x = 4$ $\qquad$ $10^x = 10$

Inverse Property of Logarithms
If b and M are positive real numbers, with $b \neq 0$, then

$$b^{\log_b M} = M$$

Evaluate

$\cancel{5^{\log_5}} 20$

20

$\cancel{8^{\log_8}} 12$

12

Evaluate

$\cancel{12^{\log_{12}}}\, 3$ $\qquad\qquad$ $\cancel{10^{\log}}\, 6$

3 $\qquad\qquad$ 6

Inverse Property of Logarithms
If b and r are positive real numbers, with $b \neq 0$, then

$$\log_a a^r = r$$

Evaluate

$$\cancel{\log_4 4^3}$$

3

$$\cancel{\ln e^7}$$

7

Evaluate

$\cancel{\log_8 8^3}$

3

$\cancel{\log 10^{-4}}$

-4

Product Rule of Logarithms
If M, N and b are positive real numbers, with $b \neq 0$, then

$$\log_b(MN) = \log_b M + \log_b N$$

Write each of the following logarithms as the sum of logarithms.

$$\log_2(5 \cdot 3)$$

$\log_2 5 + \log_2 3$

$$\ln(6z)$$

$\ln(6) + \ln(z)$

Write as a sum of logarithms

$$\log_4(9 \cdot 5)$$

$\log_4 9 + \log_4 5$

$$\log(5w)$$

$\log 5 + \log w$

quotient

~~Product~~ Rule of Logarithms

If M, N and b are positive real numbers, with $b \neq 0$, then

$$\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$$

quotient Rule

$$\log_2\left(\frac{5}{3}\right) = \log_2 5 - \log_2 3$$

$$\log\left(\frac{y}{5}\right) = \log y - \log 5$$

Write as a difference of logarithms

$$\log_7\left(\frac{9}{5}\right) \quad \log_7 9 - \log_7 5$$

$$\ln\left(\frac{p}{3}\right) \quad \ln p - \ln 3$$

Write the following as the sum or difference of logarithms.

$$\log_3\left(\frac{3m}{n}\right) \qquad\qquad \log_3\left(\frac{q}{3p}\right)$$

$$\left(\log_3 3 + \log_3 m\right) - \log_3 n$$

$$\cancel{\log_3 3 \cdot \frac{m}{n}}$$

$$\log_3\left(\frac{3m}{n}\right)$$

$$\log_3 q - \log_3(3p)$$

$$\log_3 q - \left(\log_3(3) + \log_3(p)\right)$$

Product Rule of Logarithms
If M and b are positive real numbers, with $b \neq 0$, then

$$\log_b M^r = r \log_b M$$

Use the power Rule of Logarithms to express all powers as factors.

$5\log_8 3^5$

$\ln x^3$

$5 \cdot \log_8 3$

$3 \ln x$

Use the power Rule of Logarithms to express all powers as factors.

$\log_2 5^6$ $\qquad\qquad$ $\log b^5$

$6 \cdot \log_2 5$ $\qquad\qquad$ $5 \log b$

Expand the following logarithms.

$\log_2(x^2 \cdot y^3)$

$\log\left(\frac{100 \cdot x}{y}\right)$

$\log_2 x^2 + \log_2 y^3$

$2\log_2 x + 3\log_2 y$

$(\log 100 + \log x) - \log y$

Expand the following logarithms.

$$\log_4(a^2b)$$

$\log_4 a^2 + \log_4 b$

$2\log_4 a + \log_4 b$

$$\log_3\left(\frac{9m^4}{n}\right)$$

$(\log_3 9 + 4\log_3 m) - \log_3 n$

Write each of the following as a single logarithm.

$$\log_6 3 + \log_6 12$$

$$\log_6(3 \cdot 12)$$

$$\log\boxed{(x-2)} - \log x$$

$$\log\left(\frac{(x-2)}{x}\right)$$

Write each of the following as a single logarithm.

$$\log_8 4 + \log_8 16$$

$$\log_8 (4 \cdot 16)$$

$$\log_3 (x+4) \div \log_3 (x-1)$$

$$\log_3 \frac{(x+4)}{x-1}$$

Write each of the following as a single logarithm.

$$2\log_2(x-1)+3\log_2 x$$

$$\log_2\left((x-1)^2 \cdot x^3\right)$$

$$\log(x+1)-4\log x$$

$$\log \frac{(x+1)}{x^4}$$

Change of Base Formula

If $a \neq 0$, $b \neq 0$, and M are positive real numbers, then

$$\log_a M = \frac{\log_b M}{\log_b a}$$

which means:

$$\log_a M = \frac{\log M}{\log a} = \frac{\ln M}{\ln a}$$

$\log_{\square} \square =$

Use your calculator to approximate the following:

$\log_4 45 = \dfrac{\log 45}{\log 4} = 2.74$

$\log_3 26 \quad \dfrac{\log 26}{\log 3} = 2.96$

Summary of Properties

$$\log_a a^r = r \qquad b^{\log_b M} = M$$

$$\log_b(MN) = \log_b M + \log_b N$$

$$\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$$

$$\log_b M^r = r \log_b M$$

$$\log_a M = \frac{\log_b M}{\log_b a}$$