fractions

7-1 Rational Graphs

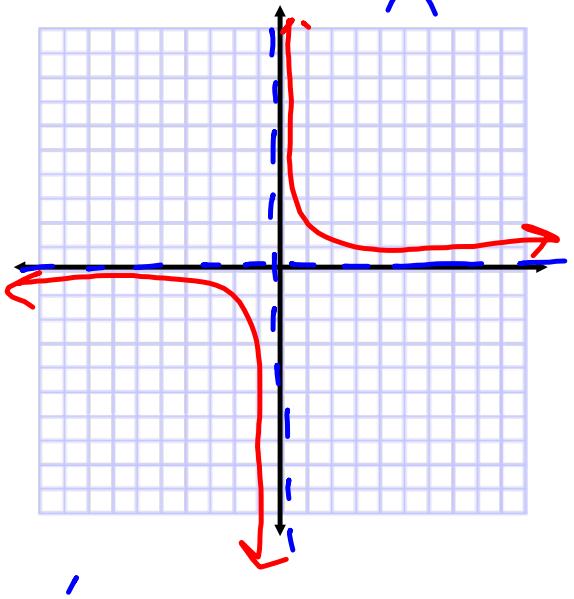
Objectives:

I can determine the domain, range, symmetry, end behavior, and intervals of increasing and decreasing of rational graphs.

I can identify the transformation of a given function and sketch a graph

I can write a rational equation given a graph.

$$f(x)=\frac{1}{x}$$ $x \neq 0$

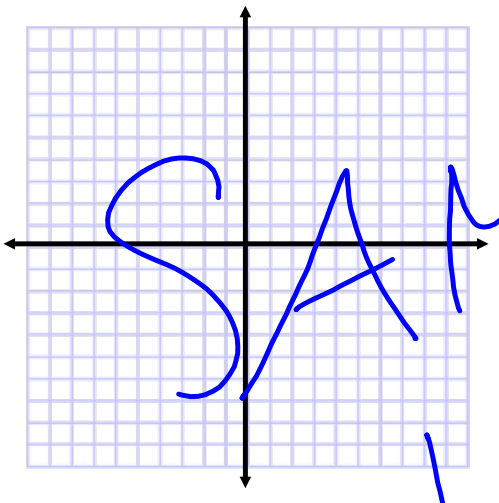


STAR

Domain $(-\infty,0)\cup(0,\infty)$
Range $(-\infty,0)\cup(0,\infty)$
Increasing DNE
Decreasing $(-\infty,0)\cup(0,\infty)$
Left End Behavior $\lim_{x\to-\infty} f(x)=0$
Right End Behavior $\lim_{x\to\infty} f(x)=\infty$
x-intercepts DNE
y-intercepts
Vertical Asymptote(s): $x=0$
Horizontal Asymptote: $y=0$
One-to-One?
yes

Rational w/ odd power

Equation: $\frac{1}{x}, \frac{1}{x^3}, \frac{1}{x^5} \ldots$



Domain
Range
Increasing
Decreasing
Left End Behavior
Right End Behavior

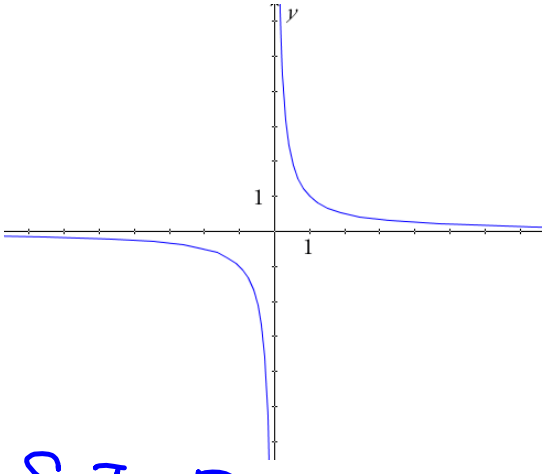
SAME as

$\frac{1}{x}$

x-intercepts
y-intercepts
Vertical Asymptote(s):
Horizontal Asymptote:
One-to-One?

Look at the following Graphs $f(x)=\frac{1}{x}$ and $f(x)=\frac{1}{x^2}$ and compare. What is going on?

$$f(x)=\frac{1}{x}$$



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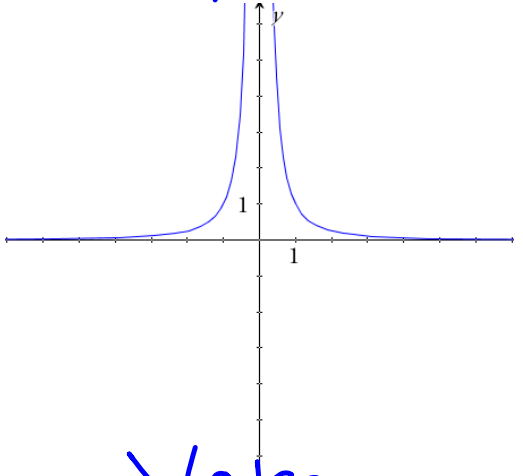
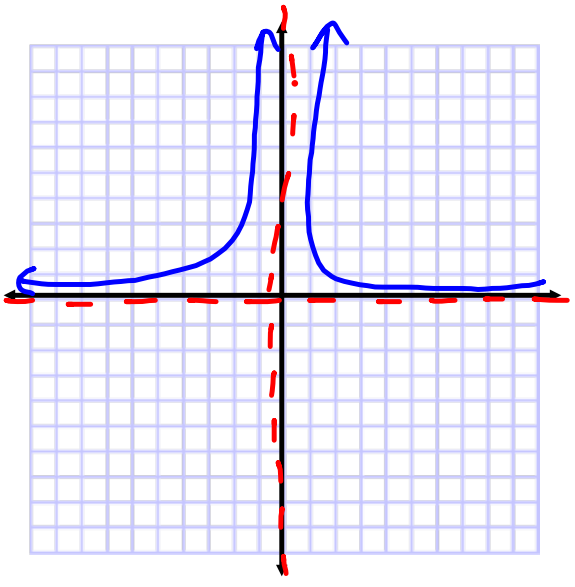
$$f(x)=\frac{1}{x^2}$$



Volcano

Rational w/even power



Equation: $\frac{1}{x^2}, \frac{1}{x^4}, \frac{1}{x^6} \ldots$

Domain $(-\infty, 0) \cup (0, \infty)$

Range $(0, \infty)$

Increasing $(-\infty, 0)$

Decreasing $(0, \infty)$

Left End Behavior $\lim_{x \to -\infty} f(x) = 0$

Right End Behavior $\lim_{x \to \infty} f(x) = 0$

x-intercepts DNE

y-intercepts

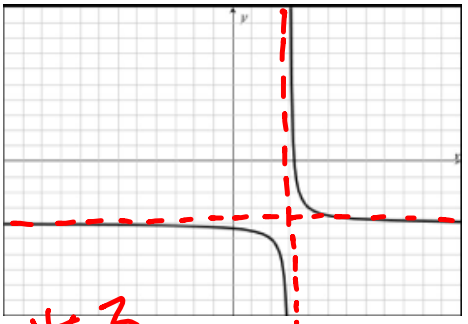
Vertical Asymptote(s): $x = 0$

Horizontal Asymptote: $y = 0$

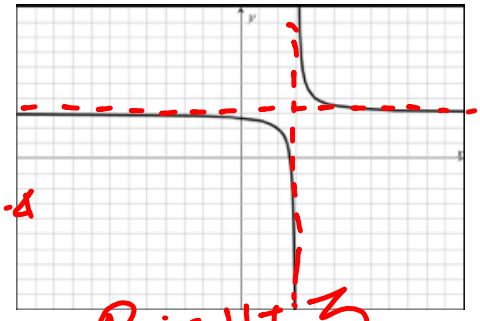
One-to-One?

No

$$f(x)=\frac{1}{x-3}-4$$

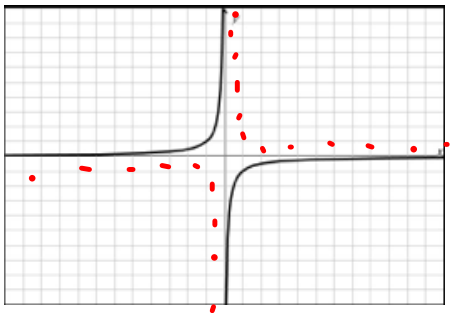


$$f(x)=\frac{1}{x-3}+3$$



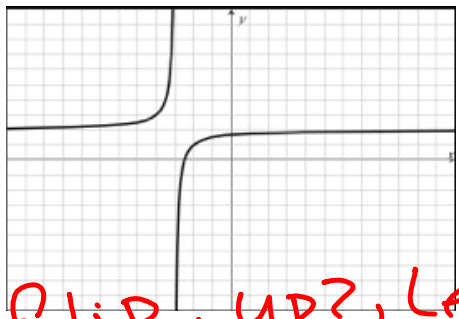
Based on the equations and corresponding graphs, what do you conclude about the transformations?

$$f(x) = -\frac{1}{x}$$



Reflect

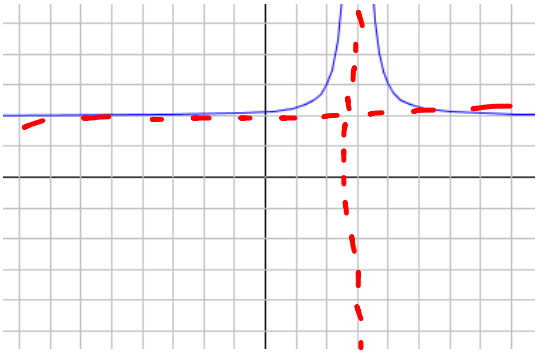
$$f(x) = -\frac{1}{x+3} + 2$$



flip, up 2, left 3

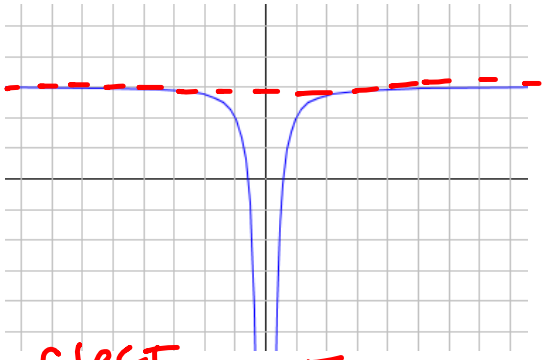
Based on the equations and corresponding graphs, what do you conclude about the transformations?

$$f(x)=\frac{1}{(x-3)^2}+2$$



R 3 up 2

$$f(x)=-\frac{1}{x^2}+3$$



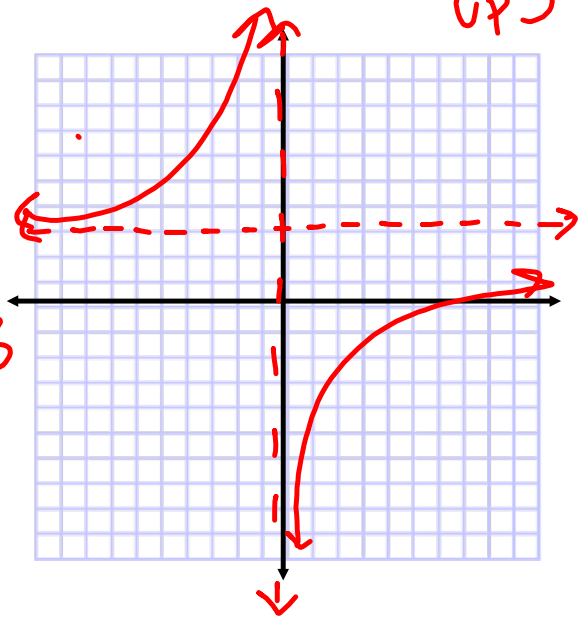
Reflect over x, up 3

Based on the equations and corresponding graphs, what do you conclude about the transformations?

Sketch a graph and analyze of the following.

$$f(x) = -\frac{1}{x} + 3$$

flip
up 3



Domain: $(-\infty, 0) \cup (0, \infty)$

Range: $(-\infty, 3) \cup (3, \infty)$

V Asymptote: $x = 0$

H Asymptote: $y = 3$

Increasing: $(-\infty, 0) \cup (0, \infty)$

Decreasing: DNE

End Behavior:

R $\lim_{x \to \infty} f(x) = 3$

L $\lim_{x \to -\infty} f(x) = 3$

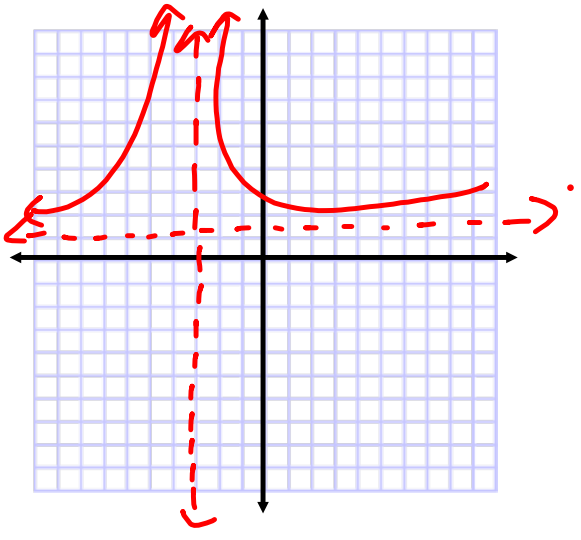
V Asymptote behavior:

$\lim_{x \to 0^+} f(x) = -\infty$

$\lim_{x \to 0^-} f(x) = \infty$

Sketch a graph and analyze of the following.

$$f(x)=\frac{1}{(x+3)^2}+1$$



Domain: $(-\infty,-3)\cup(-3,\infty)$

Range: $(-\infty,1)\cup(1,\infty)$

V Asymptote: $x=-3$

H Asymptote: $y=1$

Increasing: $(-\infty,-3)$

Decreasing: $(-3,\infty)$

End Behavior:

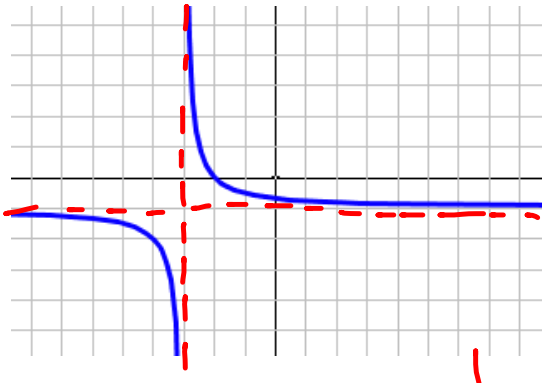
$\lim_{x\to\infty} f(x)=1$ $\lim_{x\to-\infty} f(x)=1$

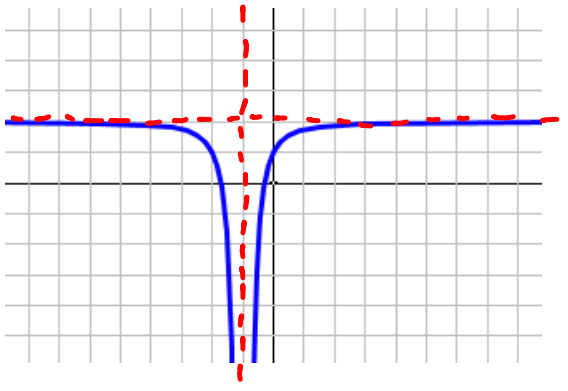
Asymptote behavior:

$\lim_{x\to-3^+} f(x)=\infty$

$\lim_{x\to-3^-} =\infty$

Based on the conclusions you made, work with a partner to write an equation based on the following graphs.





left 3
down 1

$f(x) = \frac{1}{x+3} - 1$

left 1
UP 2
Reflected over x

$f(x) = -\frac{1}{(x+1)^2} + 2$

When given a rational function in the form of $f(x)=\frac{mx+n}{px+q}$ where $m \neq 0$ and $p \neq 0$, you can use division to re-write the function in a form to identify the transformations.

$$g(x)=\frac{3x-4}{x-1}$$

1 | 3 -4
  | 3
3 | -1

$3+\frac{-1}{x-1}$

$\frac{-1}{x-1}+3$

- flip
- R 1
- up 3

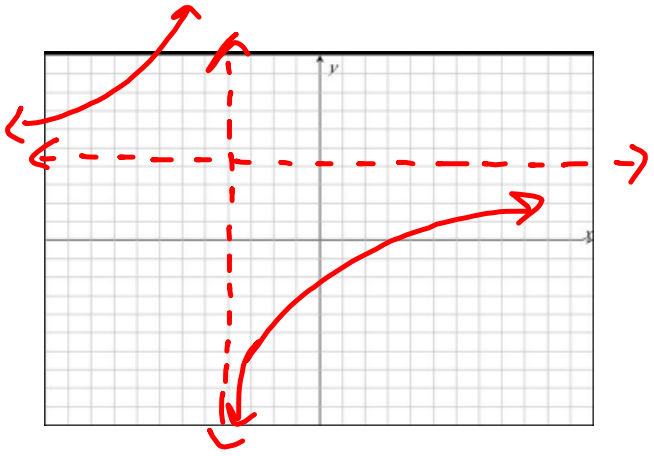
1. $\frac{1}{x}, \frac{1}{x^2}$
2. $\frac{mx+n}{px+q}$
3. $\frac{(\ x\ )}{(\ )(\ )}$

Given $f(x)=\dfrac{4x+7}{x+4}$, use division to re-write the function and identify the transformations. Then sketch a graph and state the domain, range, and intervals of increasing and decreasing.

$$\begin{array}{r|rr} -4 & 4 & 7 \\ & \downarrow & -16 \\ \hline & 4 & -9 \end{array}$$

$$4+\frac{-9}{x+4}$$

UP 4
Left 4
flip
ST 9

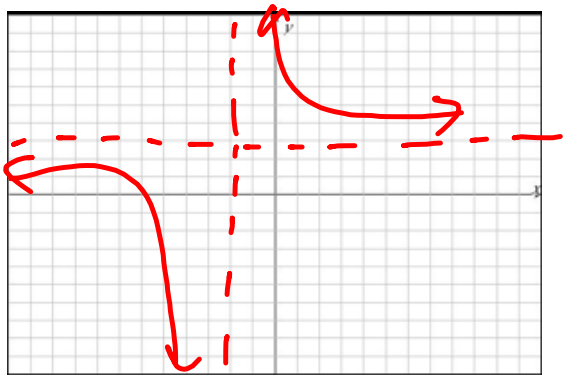


Given $f(x)=\dfrac{3x+7}{x+2}$, use division to re-write the function and identify the transformations. Then sketch a graph and analyze.

$$-2\,\big]\;3 \quad 7$$
$$\downarrow \quad -6$$
$$3 \quad \big[1$$

$$3+\frac{1}{x+2}$$

up 3
left 2



$$f(x)=\frac{5-2x}{x+4}$$