6-4 Inverse Functions

Objectives:
-I can find the inverse of a given function graphically and algebraically

Inverse of a Relation

The **inverse of a relation** consisting of the ordered pairs (x, y) is the set of all ordered pairs (y, x).

Notation:

$$f^{-1}(x)$$

Represents the inverse of the function $f(x)$

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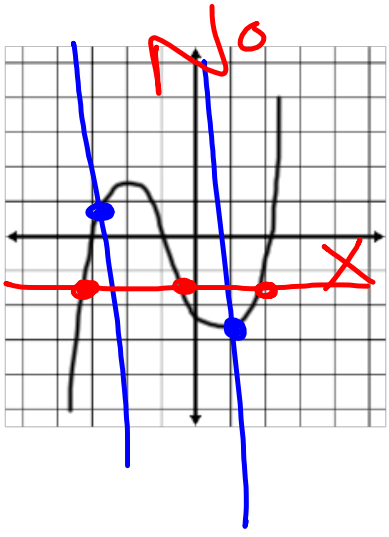
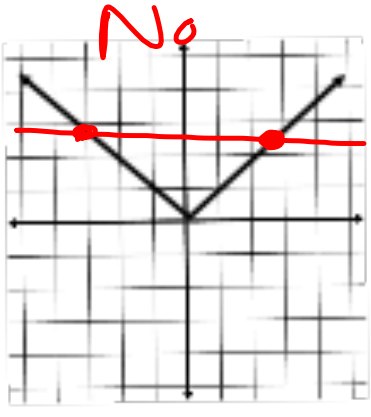
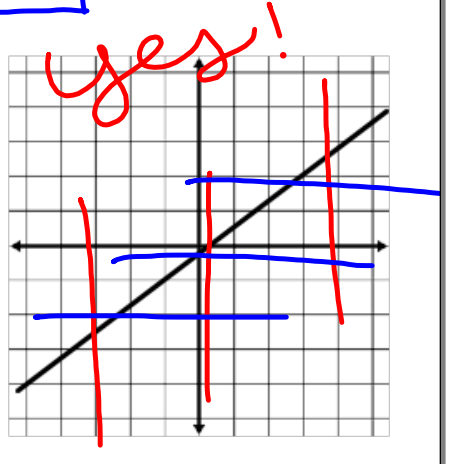
Horizontal-Line Test

The inverse of a function is a function if and only if every horizontal line intersects the graph of the given function (passed the vertical-line test) at no more than one point.

If a function passes both the vertical line test AND the horizontal line test, then it is a **one-to-one** function.

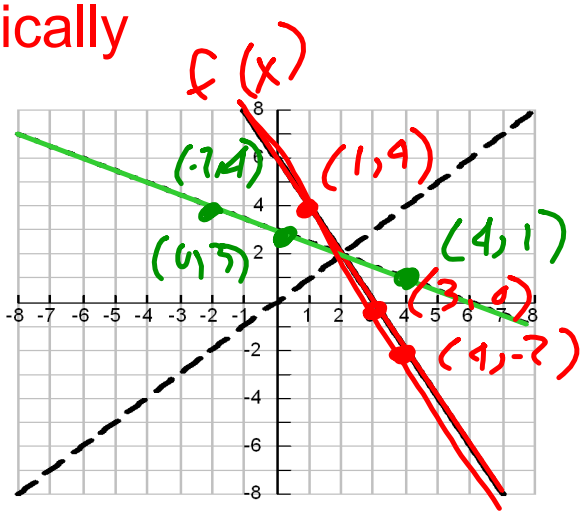






Inverses - graphically

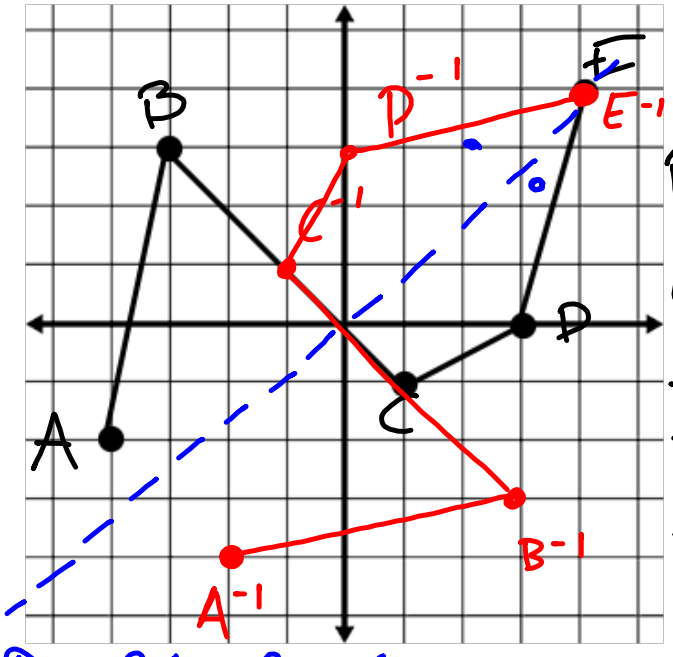
Show $f(x) = 6 - 2x$ and $g(x) = \frac{6 - x}{2}$ are inverses graphically.



f(x): (1,4) (3,0) (4,-2)

g(x): (4,1) (0,3) (-2,4)

Graph the inverse of the graph. (Use y=x to find inverse points)

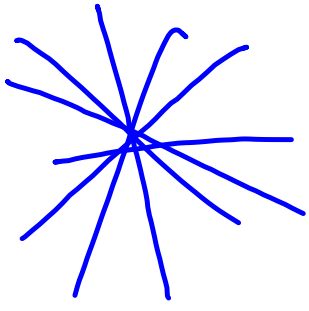


Reflect over $y=x$

$f(x)$	$f^{-1}(x)$
$A(-4,-2)$	$A^{-1}(-2,-4)$
$B(-3,3)$	$B^{-1}(3,-3)$
$C(1,-1)$	$C^{-1}(-1,1)$
$D(3,0)$	$D^{-1}(0,3)$
$E(4,4)$	$E^{-1}(4,4)$

To find the inverse equation of a function

1. Change $f(x)$ to y.
2. Interchange x and y
3. Solve for y
4. Change new y to $f^{1}(x)$



Find the inverse of each function

$f(x) = x^2 + 1$

$y = x^2 + 1$

$x = y^2 + 1$

$-1 \quad -1$

$\sqrt{x-1} = \sqrt{y^2}$

$y = \sqrt{x-1}$

$f^{-1}(x) = \sqrt{x-1}$

$g(x) = \sqrt[3]{x} - 3$

$y = \sqrt[3]{x} - 3$

$x = \sqrt[3]{y} - 3$

$+3 \quad +3$

$(x+3)^3 = (\sqrt[3]{y})^3$

$y = (x+3)^3$

$g^{-1}(x) = (x+3)^3$

$h(x) = 2x^3 + 3$

$y = 2x^3 + 3$

$x = 2y^3 + 3$

$-3 \quad -3$

$\frac{x-3}{2} = \frac{2y^3}{2}$

$\sqrt[3]{\frac{x-3}{2}} = \sqrt[3]{y^3}$

$\sqrt[3]{\frac{x-3}{2}} = y \quad h^{-1}(x)$

$k(x) = \frac{x+1}{2x+3}$

$y = \frac{x+1}{2x+3}$

$(2y+3)x = \frac{y+1}{2y+3}(2y+3)$

$x(2y+3) = y+1$

$2xy + 3x = y + 1$

$-3x \quad -y \quad -3x \quad -y$

$2xy - y = -3x + 1$

$y(2x-1) = -3x+1$

$\frac{y(2x-1)}{2x-1} = \frac{-3x+1}{2x-1}$

$k^{-1}(x) \; y = \frac{-3x+1}{2x-1}$