6-4 Exponential Review

Objectives:

-I can apply exponential properties and use them

-I can model real-world situations using exponential functions

EXPONENTIAL FUNCTION

$$f(x) = a(b)^x$$ ← Exponent

Initial Value
(y-intercept)

0th term

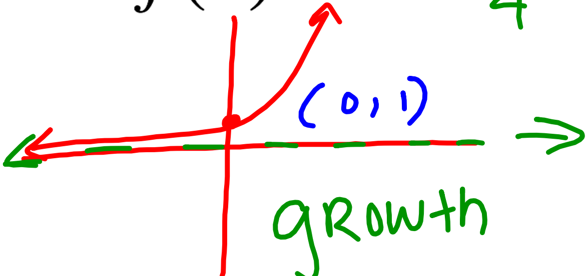
Base
(Multiplier)

r

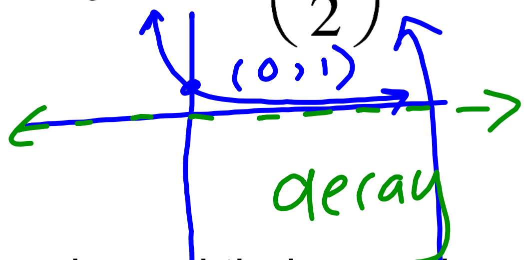
common
factor

Graph the following functions on a calculator and sketch.
Be sure to plot the y-intercept

a. $f(x) = 2^x$ $\quad \frac{7}{4}^{x}$



b. $f(x) = \left(\frac{1}{2}\right)^x$



What did you notice about the graphs and their equations?

opposite ways

x as exponent

cross @ same point

Multiply by a fraction

above x axis = asymptote

$$f(x) = a(b)^x$$

Exponential Growth and Decay

When b>1, the function represents **exponential growth**

When 0<b<1, the function represents **exponential decay**

b > 1 = GROWTH

b < 1 = decay

Determine whether each function represents growth or decay

$a \cdot b^x$

a. $f(x) = 13\left(\frac{1}{3}\right)^x$

decay

$a \cdot b^x$

b. $g(x) = 1\left(\frac{3}{2}\right)^x$

growth

general

Growth/Decay Equation

Final amount

growth

Time

$$f(t) = a(1 \pm r)^t$$

decay

Rate of change

5%

$5 \div 100 = .05$

initial amount

(y-int)

AS A DECIMAL

.05%

The population of Orem in 1950 was 4,000 and was increasing at a rate of 2.6% per year.

a) Predict the population of Orem in 1975 and 2000.

$f(t) = 4000(1+.026)^t$

1975

$f(t) = 4000(1.026)^{25} = 7,599$

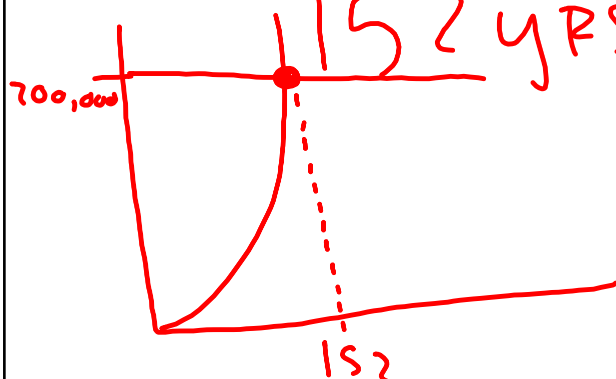
2000

$f(t) = 4000(1.026)^{50} = 14,435$

b) Using your graphing calculator, predict when Orem's population will hit 200,000 people.

$200,000 = 4000(1.026)^t$

152 yrs



$$\begin{array}{r} 1950 \\ 152 \\ \hline 2102 \end{array}$$

On federal income tax returns, self employed people can depreciate the value of business equipment. Suppose a computer valued at \$2765 depreciates at a rate of 30% per year.

$$f(t) = a(1 \pm r)^t$$

a) Write an exponential equation to model this situation

$$f(t) = 2765(1-.3)^t$$

b) How much will this computer be worth in 5 years?

$$f(5) = 2765(.7)^5 = 465\$$$

c) Use your graphing calculator to determine in how many years will the computer be worth \$350.

$$\$350 = 2765(.7)^t$$

ymax

$\boxed{5.79\text{ yrs}}$

Compound Interest Formula

$$A(t) = P\left(1+\frac{r}{n}\right)^{nt}$$

P is the principal (initial amt)

r is the annual interest rate as a decimal!

n is the number of compounding periods per year

t is the time in years

12 = monthly
4 = quarterly
1 = annually
2 = semi-annually
~~1/2~~ = bi-annually
52 = weekly
365 = daily

Write an equation then find the final amount for each investment.

$$A(t) = P\left(1+\frac{r}{n}\right)^{nt}$$

a. \$1000 at 8% compounded semiannually for 15 years

$$A(15) = 1000\left(1+\frac{.08}{2}\right)^{2\cdot 15} = \$3,243$$

Using a calculator, determine how many years it will take for equation b to have a final amount of \$4000.

$$4000 = 100\left(1+\frac{.08}{2}\right)^{2t}$$

$$\boxed{17.6 \text{ YRS}}$$

The value e is called the natural base

The exponential function with base e, $f(x)=e^x$, is called the natural exponential function.

$$e \approx 2.71828182827$$

what you need to know is $e \approx 2.7$

Evaluate $f(x) = e^x$ for

a. $x = 2$

b. $x = \frac{1}{2}$

c. $x = -1$

Continuous Compounding Formula

If P dollars are invested at an interest rate r, that is compounded continuously, then the amount, A, of the investment at time t is given by

Rate as decimal

$$\underline{A(t) = Pe^{rt}} \leftarrow \text{time}$$

final amt

principal

A person invests \$1550 in an account that earns 4% annual interest compounded continuously. $A(t) = Pe^{rt}$

a. Write an equation to represent this situation

$$A(t) = 1550e^{.04t}$$

b. Using a calculator, find when the value of the investment reaches \$2000.

?

$$2000 = 1550e^{.04t}$$

An investment of $1000 earns an annual interest rate of 7.6%.

Compare the final amounts after 8 years for interest *compounded quarterly* and for interest *compounded continuously.*