5-3: Graphing Rational Functions

$\frac{1}{x}$ $\frac{2x+5}{x+7}$ $\frac{}{x^2+\dots}$

Objectives:

1. I can find the x and y intercepts of a rational function
2. I can find the vertical and horizontal asymptotes of a rational function
3. I can find the holes of a rational function
4. I can analyze a graph of a rational function
5. I can graph a rational function by hand

X and Y Intercepts

Y intercepts, x = 0

$$f(x) = \frac{3x-12}{x^2-5x-6}$$

(0, 2)

- where x = 0
- change x to 0 and solve
- coordinate (0, y)

$$y = \frac{3(0)-12}{0^2-5(0)-6} = \frac{-12}{-6} = 2$$

X intercepts, y = 0

$$f(x) = \frac{3x-12}{x^2-5x-6}$$

$$0 = \frac{3x-12}{x^2-5x-6}$$

$$0 = 3x - 12$$

$$+12 \quad +12$$

$$\frac{12}{3} = \frac{3x}{3}$$

$$x = 4$$

(4, 0)

- where y = 0
- change y to 0 and solve
- coordinate (x, 0)
- denominator doesn't matter

Find the x and y intercepts of the following functions:

$$f(x)=\frac{(x-3)(x+1)}{x+2}$$

$$f(x)=\frac{3x-5}{(x-2)(x-3)}$$

x-int: a s

$0=\frac{(x-3)(x+1)}{x+2}$ $x=3,-1$

(3,0)(-1,0)

y-int:

$y=\frac{(0-3)(0+1)}{0+2}=\frac{-3\cdot 1}{2}=-\frac{3}{2}$ $\left(0,-\frac{3}{2}\right)$

Review of Vertical Asymptotes

$$f(x) = \frac{2}{x+3}$$

$X = -3$

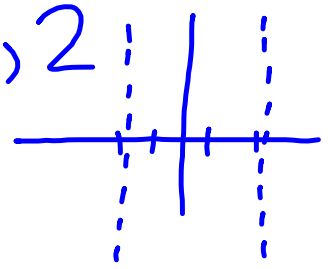
Set the denominator = 0, then solve for x

- write as $X =$
- excluded values
- Makes denominator $= 0$
-

Find the vertical asymptotes:

a. $y = \dfrac{3x-5}{(x-2)(x+2)}$

$x = -2, 2$



b. $y = \dfrac{2x^3}{x-5}$

$x = 5$

c. $y = \dfrac{5x}{x+2}$

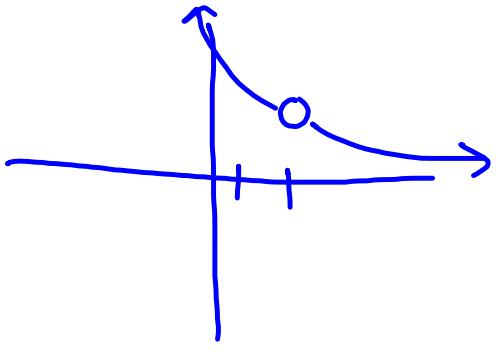
$x = -2$

~~Asymptotes:~~ Holes

check for holes before VA!! (by reducing the fraction if possible)

$$f(x)=\frac{(x-3)(x-2)}{(x-2)}$$ Hole @ x=2

- would be asymptotes BUT
- matching factors on top & bottom
- write x=



ertical (VA): caused by dividing by 0

the graph approaches $-\infty$ *or* ∞

on each side of the asymptote

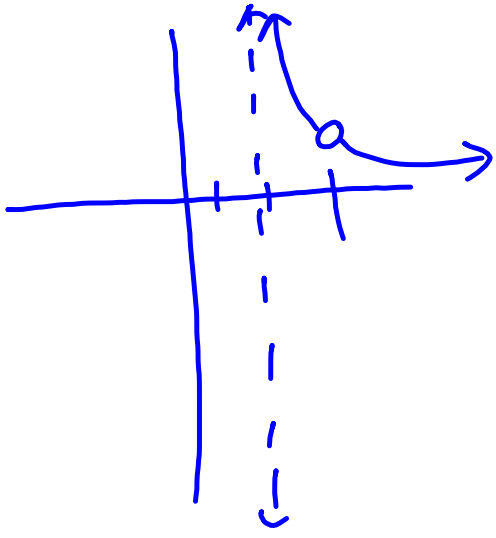
find the asymptote set den = 0 and solve

Identify any holes, then find all vertical asymptotes

$$f(x)=\frac{(x-3)(x+3)}{(x-2)(x-3)}$$

Hole : $x=3$

asymp : $x=2$

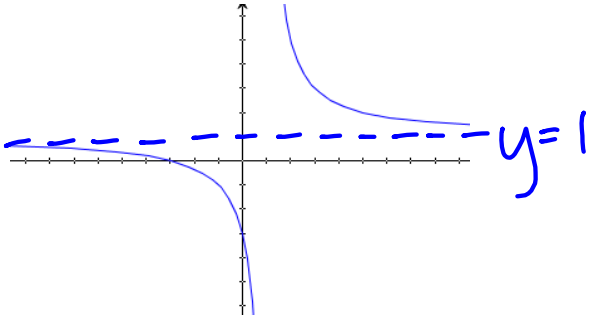


Horizontal Asymptotes

Look at the graphs, see if you can find the horizontal asymptote. Are there any patterns?

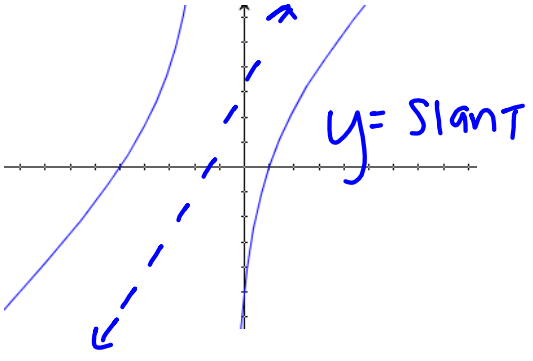
$$f(x)=\frac{1x+3}{1x-1}$$

equal degree



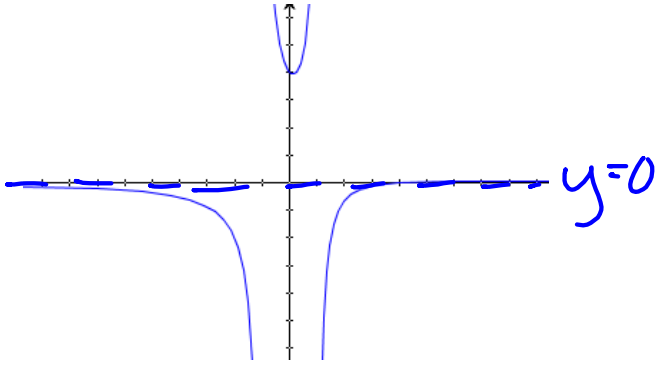
$$f(x)=\frac{(x+5)(x-1)}{x+1}$$

Top Heavy



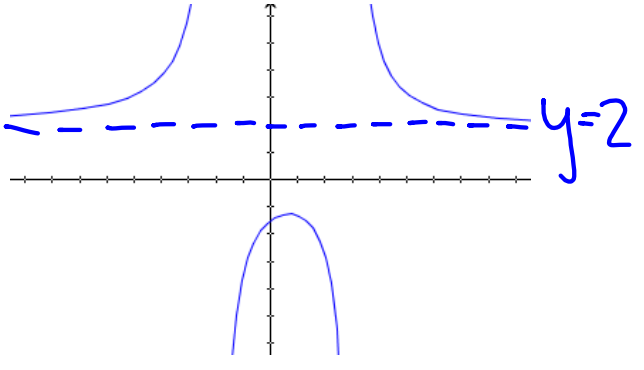
$$f(x)=\frac{x-4}{(x+1)(x-1)}$$

Bottom Heavy



$$f(x)=\frac{2x^2+9}{1x^2-x-6}$$

equal degree



Top Heavy: $y =$ slant $\quad \frac{x^3}{x}$

Bottom Heavy: $y = 0$ $\quad \frac{x}{x^2}$

Equal deg: $y =$ DIVIDE LEAD COEFFICIENTS $\quad \frac{2x}{x}, \frac{3x^2+5}{x^2-x}$

$y = 2 \qquad y = 3$

HORIZONTAL ASYMPTOTES

end behavior: (horizontal (HA) or oblique (OA)):

to find the asymptote - compare the degrees of the numerator and denominator if:

top heavy (OA):
bottom heavy (HEB): y = 0
equal (HA): divide coefficients

$x \to \infty$ $y \to$ Horizontal asymptote
$x \to -\infty$ $y \to$ Horizontal asymptote

Ex

Bottom Heavy H.A.

$\frac{(x+5)}{(x+1)(x-3)}$, $y=0$ $x \to \infty$ $y \to 0$
$x \to -\infty$ $y \to 0$

Identify the x and y intercepts, vertical and horizontal asymptotes, end behavior, and then graph.

$y = f(x) = \dfrac{-3}{x-1}$

Xint - DNE

$0 = \dfrac{-3}{x-1}$

yint - (0,3)

$y = \dfrac{-3}{0-1} = 3$

VA $x = 1$

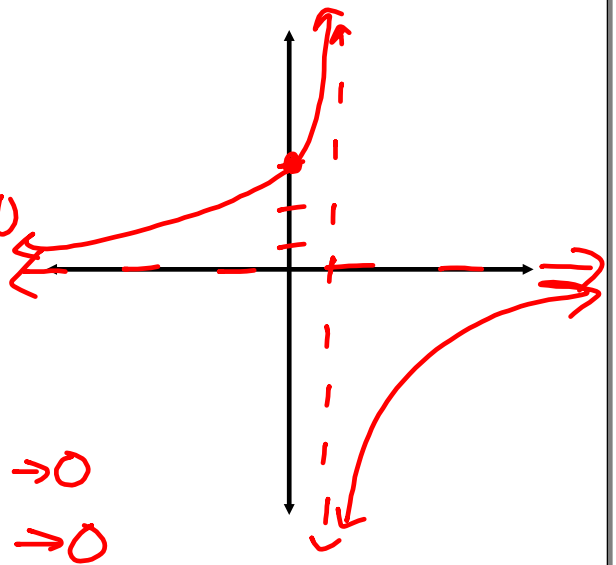
HA: $y = 0$

Bottom Heavy

EB:

$x \to \infty \quad y \to 0$

$x \to -\infty \quad y \to 0$



Holes: None

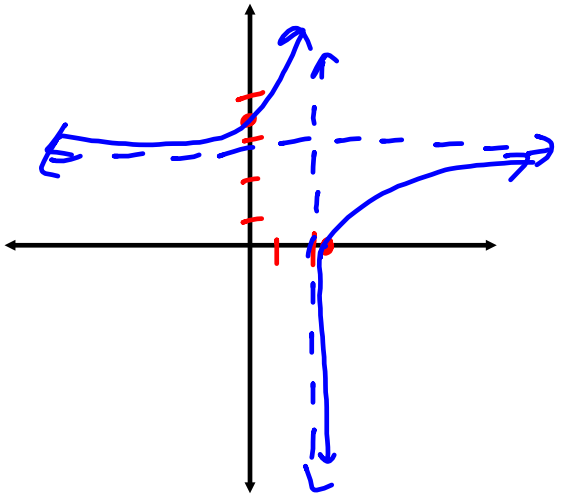
$y = f(x) = \dfrac{3x-7}{1x-2}$

Xint: (2.3, 0)

$0 = 3x - 7 \quad 7 = 3x \quad x = 7/3$

yint: (0, 3.5)

$y = \dfrac{3(0)-7}{0-2} = \dfrac{-7}{-2} = 3.5$



2 | 3 -7

VA: $x = 2$

Holes: DNE

HA: equal deg $y = 3$

EB

$x \to \infty \quad y \to 3$

$x \to -\infty \quad y \to 3$

Identify the x and y intercepts, vertical and horizontal asymptotes, end behavior, and then graph.

$y = f(x) = \dfrac{3x-2}{x-1}$

x int (.6, 0)

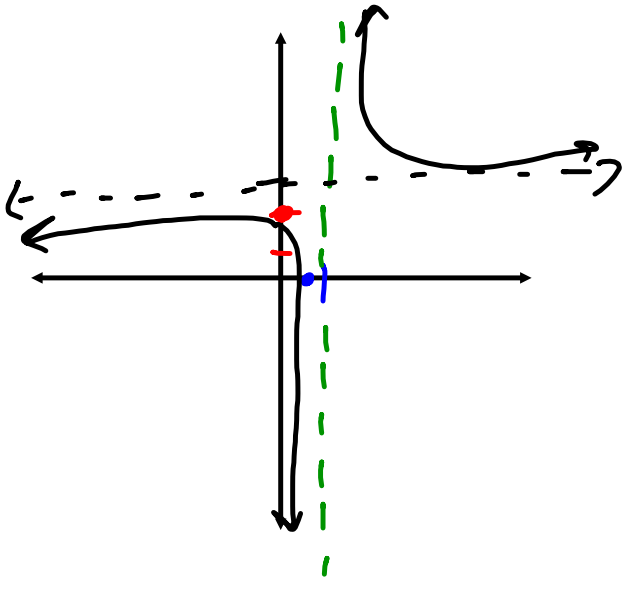
$0 = 3x - 2 \quad 2 = 3x \quad x = \frac{2}{3}$

y int (0, 2)

$y = \dfrac{3(0)-2}{0-1} = \dfrac{-2}{-1} = 2$

VA $x = 1$

Holes: DNE



HA: = deg $y = \frac{3}{1} = 3$

EB: $x \to \infty \quad y \to 3$

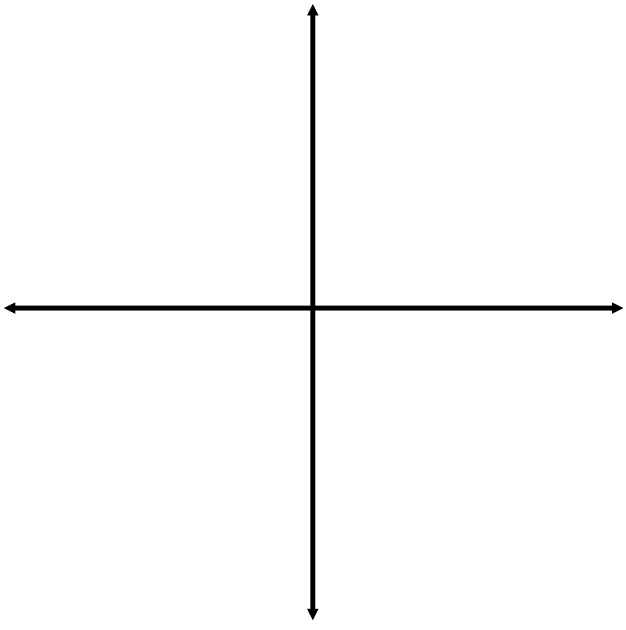
$x \to -\infty \quad y \to 3$

Find the intercepts, asymptotes, limits at vertical asymptotes, analyze and draw the graph of

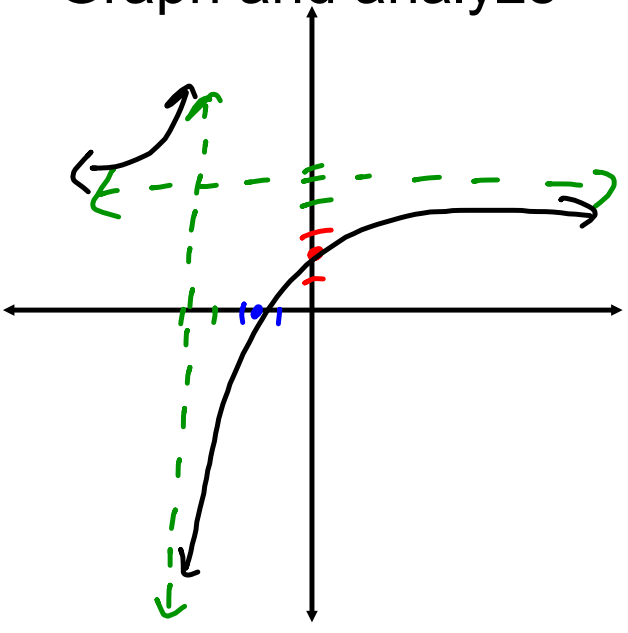
$$f(x)=\frac{x-1}{(x-4)(x+3)}$$

Domain
Range
x-intercepts
y-intercepts
VA
HA
Asymptote Behavior

End Behavior



Graph and analyze $f(x)=\dfrac{4x+7}{x+4}$



Domain
Range
x-intercepts
y-intercepts
VA
HA

Asymptote Behavior

End Behavior

xint $(-1.75, 0)$

$0=\dfrac{4x+7}{\cancel{\quad}} \quad \dfrac{-7}{4}=\dfrac{4x}{4} \quad x=-\dfrac{7}{4}$

yint $(0, 1.75)$

$y=\dfrac{4(0)+7}{0+4}=\dfrac{7}{4}=1.75$

Holes: DNE

VA: $x=-4$

HA: = deg $y=\dfrac{4}{1}=4$

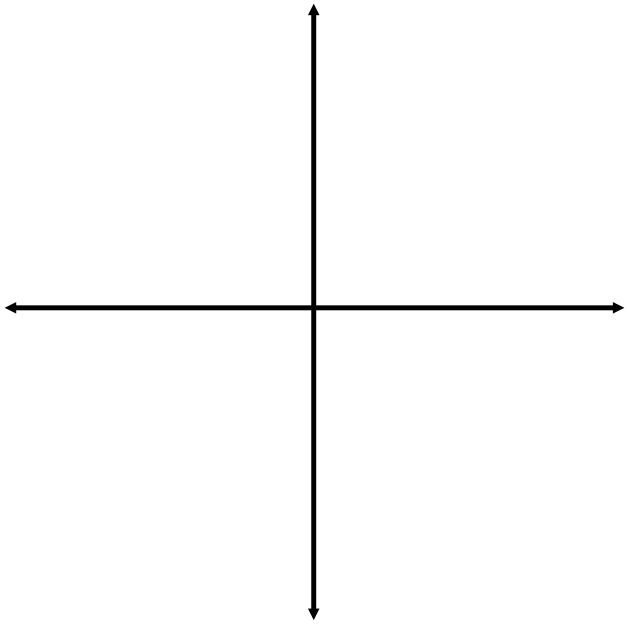
EB $x\to\infty \; y\to 4$

$x\to-\infty \; y\to 4$

Graph and Analyze $f(x)=\dfrac{x+1}{(x+3)(x-4)}$

Domain
Range
x-intercepts
y-intercepts
VA
HA

Asymptote Behavior



End Behavior

Graph and analyze $f(x)=\dfrac{4x-4}{x+2}$

Domain
Range
x-intercepts
y-intercepts
VA
HA

Asymptote Behavior

End Behavior