5-1 Rational Functions

Objectives: fraction

- I can determine the domain, range, end behavior, and intervals of increasing and decreasing of rational functions. (x's lie) (up/down)
- I can identify the transformation of a given function and sketch a graph
- I can write a rational equation given a graph.

State the domain of $f(x) = \frac{1}{x}$.

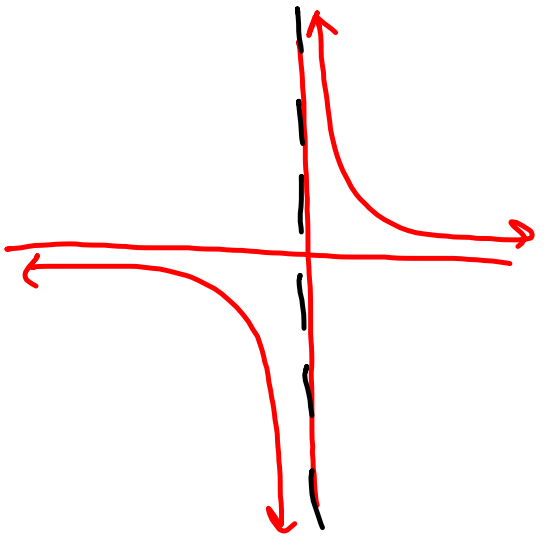
The function accepts all real numbers except 0, because division by 0 is undefined. So, the function's domain is as follows:

- ~~As an inequality: $x <$ [ ] or $x >$ [ ]~~
- ~~In set notation: $\{x \mid x \neq$ [ ] $\}$~~
- In interval notation (where the symbol $\cup$ means *union*):

$$(-\infty, 0) \cup (0, +\infty)$$

$y = \frac{1}{x}$

$x \neq 0$



Determine the end behavior of $f(x) = \frac{1}{x}$.

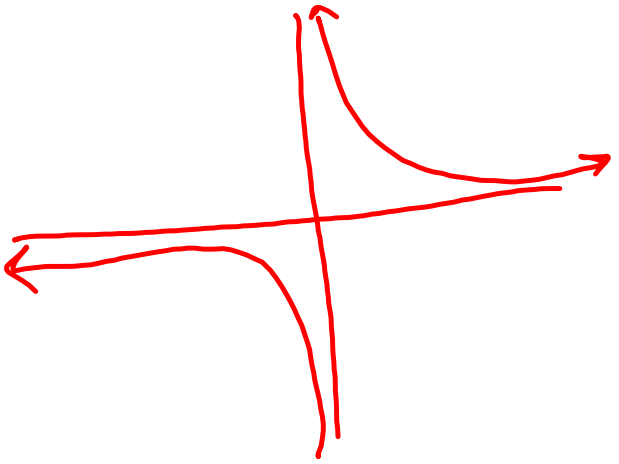
First, complete the tables.

x Increases without Bound	
x	$f(x) = \frac{1}{x}$
100	.01
1000	.001
10,000	.0001

x Decreases without Bound	
x	$f(x) = \frac{1}{x}$
−100	−.01
−1000	−.001
−10,000	−.0001

Next, summarize the results.

- As $x \to +\infty$, $f(x) \to$ 0.
- As $x \to -\infty$, $f(x) \to$ 0.



Examine the behavior of $f(x) = \frac{1}{x}$ near $x = 0$, and determine what this means for the graph of the function.

asymptote behavior

First, complete the tables.

x Approaches 0 from the Positive Direction	
x	$f(x) = \frac{1}{x}$
0.01	100
0.001	1000
0.0001	10,000

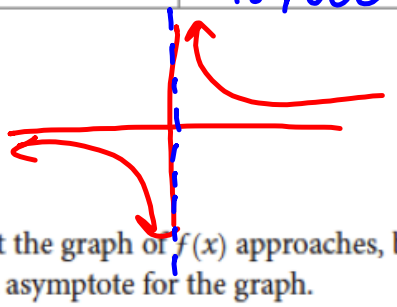
x Approaches 0 from the Negative Direction	
x	$f(x) = \frac{1}{x}$
−0.01	−100
−0.001	−1000
−0.0001	−10,000

Next, summarize the results.

Right

- As $x \to 0^+$, $f(x) \to$ ∞.
- As $x \to 0^-$, $f(x) \to$ $-\infty$.

Left

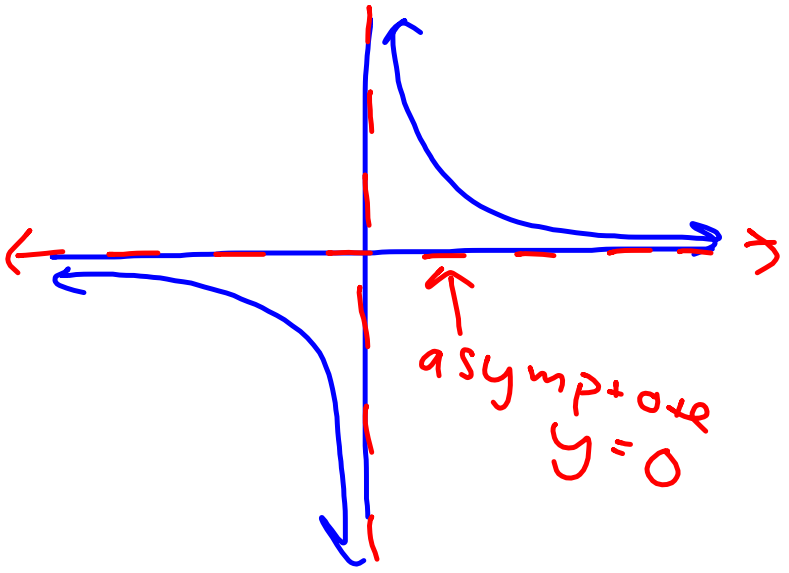


The behavior of $f(x) = \frac{1}{x}$ near $x = 0$ indicates that the graph of $f(x)$ approaches, but does not cross, the [x-axis/y-axis], so that axis is also an asymptote for the graph.

State the range of $f(x) = \frac{1}{x}$.

The function takes on all real numbers except 0, so the function's range is as follows:

- ~~As an inequality: $y <$ [ ] or $y >$ [ ]~~
- ~~In set notation: $\{y | y \neq$ [ ] $\}$~~
- In interval notation (where the symbol $\cup$ means *union*): $(-\infty, 0) \cup (0, +\infty)$

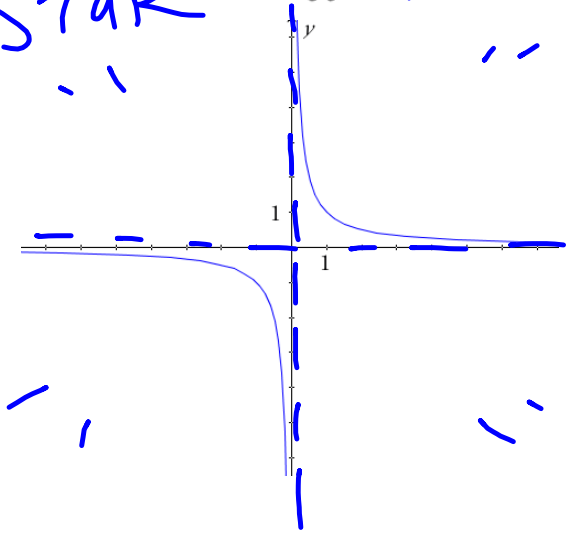


$0 = \frac{1}{x}$

Look at the following Graphs $f(x)=\frac{1}{x}$ and $f(x)=\frac{1}{x^2}$ and compare. What is going on?

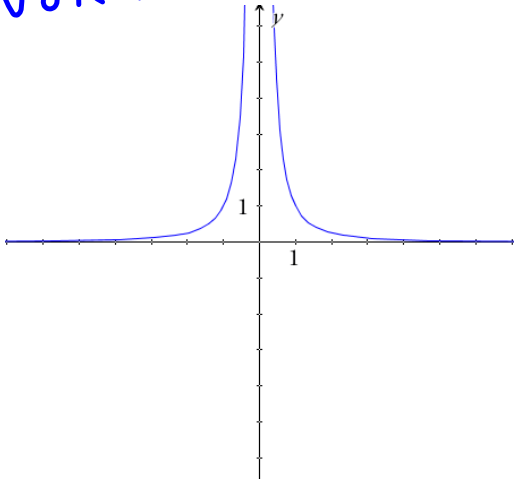
$$f(x)=\frac{1}{x}$$

STAR



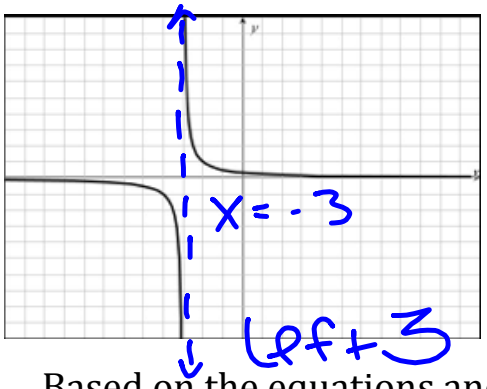
$$f(x)=\frac{1}{x^2}$$

VOLCANO

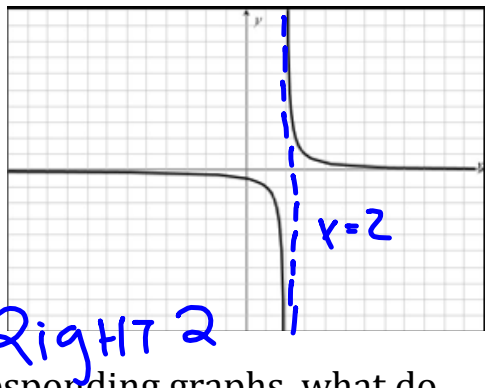


Look at the following graphs and the parent function from your function booklet and answer the question below.

$$f(x)=\frac{1}{x+3}$$



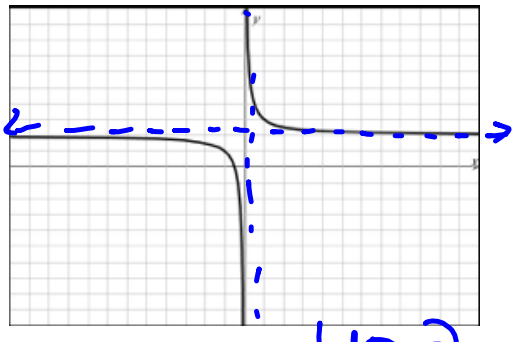
$$f(x)=\frac{1}{x-2}$$



Based on the equations and corresponding graphs, what do you conclude about the transformations?

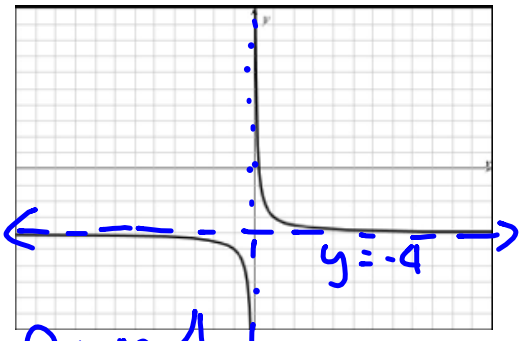
X's lie

$$f(x)=\frac{1}{x}+2$$



Up 2

$$f(x)=\frac{1}{x}-4$$

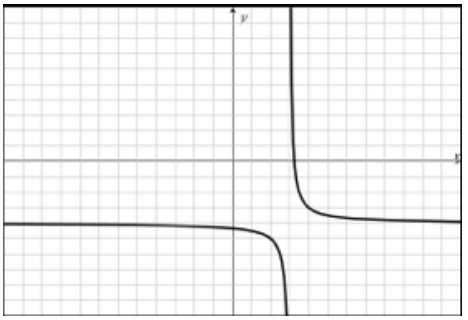


Down 4

Based on the equations and corresponding graphs, what do you conclude about the transformations?

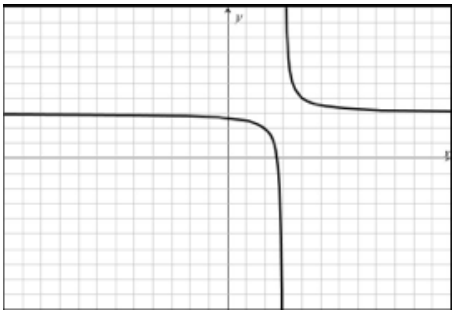
OUTSIDE: up or down

$$f(x)=\frac{1}{x-3}-4$$



down 4, Right 3

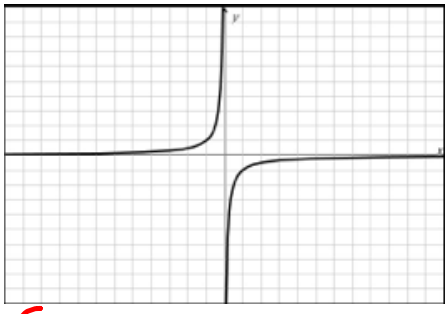
$$f(x)=\frac{1}{x-3}+3$$



Right 3, Up 3

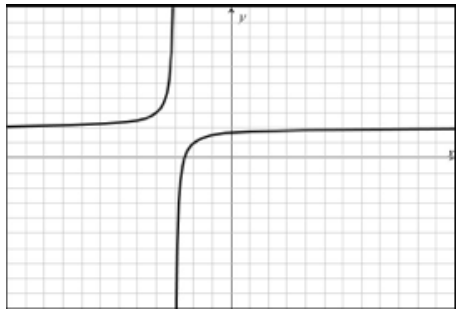
Based on the equations and corresponding graphs, what do you conclude about the transformations?

$$f(x) = -\frac{1}{x}$$



flip

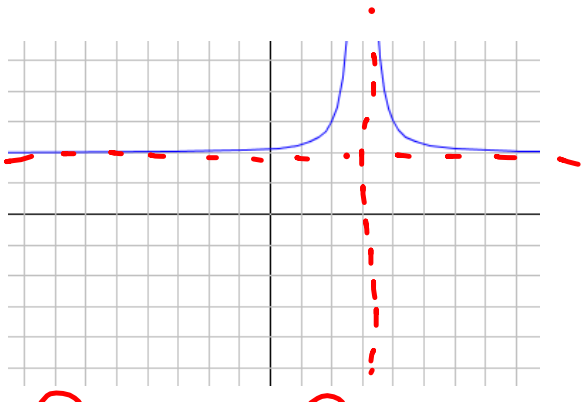
$$f(x) = -\frac{1}{x+3} + 2$$



flip, left 3, up 2

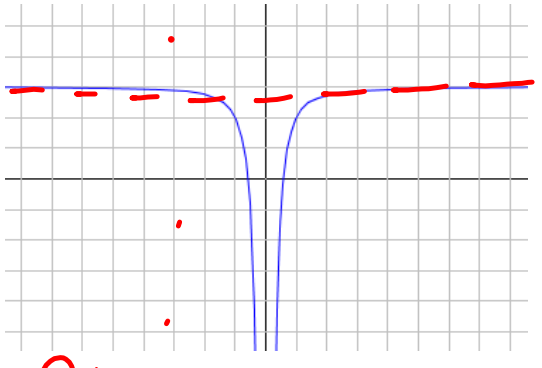
Based on the equations and corresponding graphs, what do you conclude about the transformations?

$$f(x)=\frac{1}{(x-3)^2}+2$$



Right 3, up 2

$$f(x)=-\frac{1}{x^2}+3$$

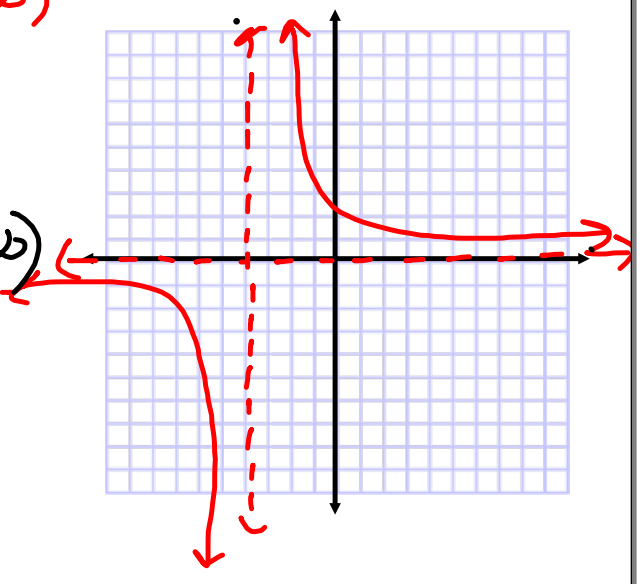


flip, up 3

Based on the equations and corresponding graphs, what do you conclude about the transformations?

Sketch a graph and analyze of the following.

$$f(x) = \frac{1}{x+4}$$



Domain: $(-\infty, -4) \cup (-4, \infty)$

Range: $(-\infty, 0) \cup (0, \infty)$

V Asymptote: $x = -4$

H Asymptote: $y = 0$

Increasing: DNE

Decreasing: $(-\infty, -4) \cup (-4, \infty)$

End Behavior:

$x \to \infty \quad y \to 0$

$x \to -\infty \quad y \to 0$

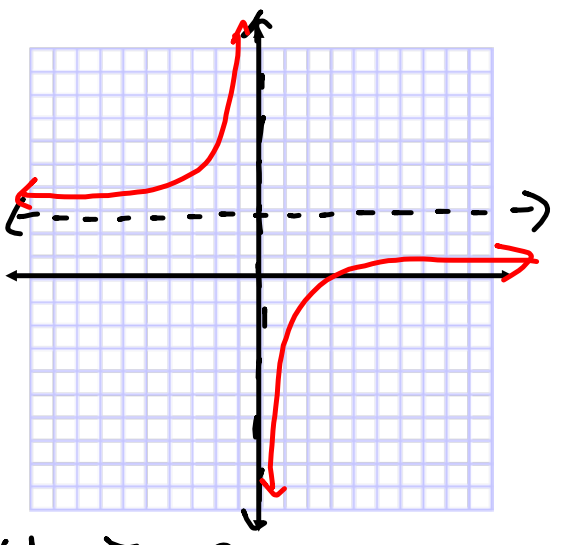
V. Asymptote behavior:

$x \to -4^{+} \quad y \to \infty$

$x \to -4^{-} \quad y \to -\infty$

Sketch a graph and analyze of the following.

$$f(x) = -\frac{1}{x} + 3$$



Domain: $(-\infty, 0) \cup (0, \infty)$

Range: $(-\infty, 3) \cup (3, \infty)$

V Asymptote: $x = 0$

H Asymptote: $y = 3$

Increasing: $(-\infty, 0) \cup (0, \infty)$

Decreasing: DNE

End Behavior:

$x \to \infty \quad y \to 3$

$x \to -\infty \quad y \to 3$

V. Asymptote behavior:

$x \to 0^+ \quad y \to -\infty$

$x \to 0^- \quad y \to \infty$

Up 3, flip

STAR

Sketch a graph and analyze of the following.

$$f(x)=\frac{1}{(x+3)^2}+1$$

Domain: $(-\infty,-3)\cup(-3,\infty)$

Range: $(1,\infty)$

V Asymptote: $x=-3$

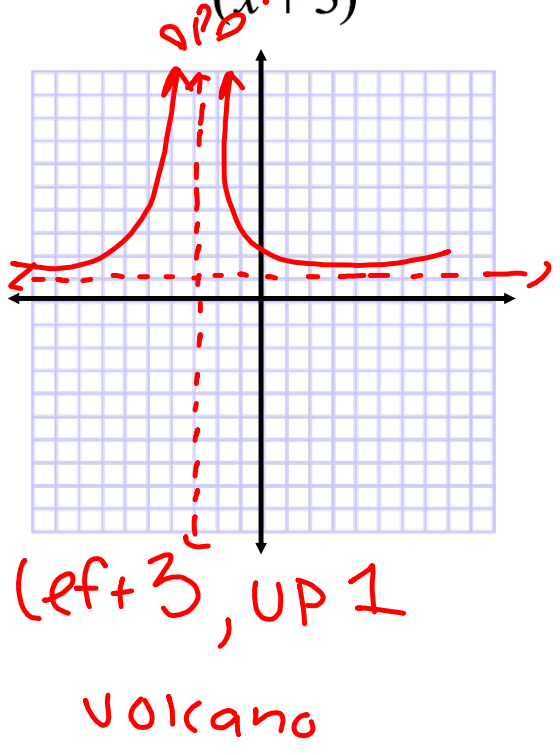
H Asymptote: $y=1$

Increasing: $(-\infty,-3)$

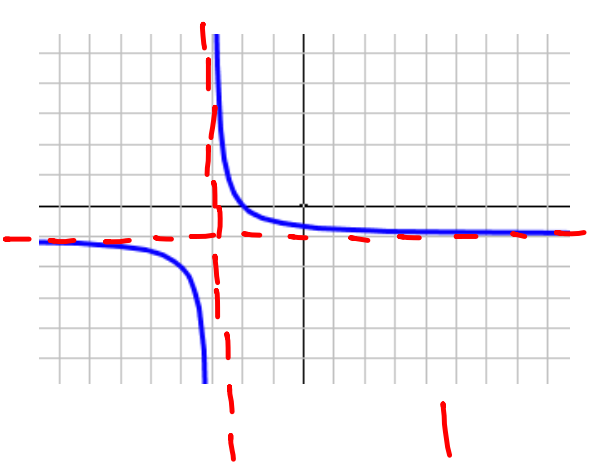
Decreasing: $(-3,\infty)$

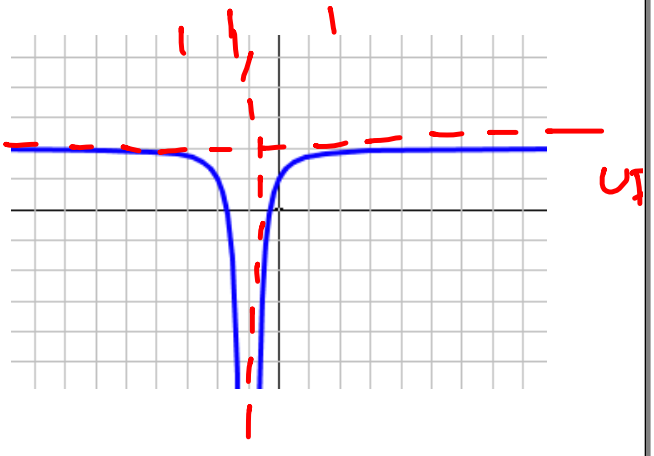
End Behavior:

Asymptote behavior:



Based on the conclusions you made, work with a partner to write an equation based on the following graphs.





$y = \frac{1}{x+3} - 1$

left 3 up 2

down 1 left 1