4.2 Complex Zeros

- I can find all zeros of a polynomial including non-real complex zeros
- I can write a polynomial from its zeros (factors)
- I can do a linear factorization written as factors

$x \wedge 1$

$a + bi$

$4 + 0i = 4$

Fundamental Thm of Alg: an nth degree polynomial will have n complex zeros

TOTAL

$x^2 \rightarrow 2$ zeros $x^5 \rightarrow 5$ zeros

(May be a combination of real and non-real complex.
Some zeros may be repeated)

$x^3 \rightarrow (x+1)^2(x-2)$ imaginary

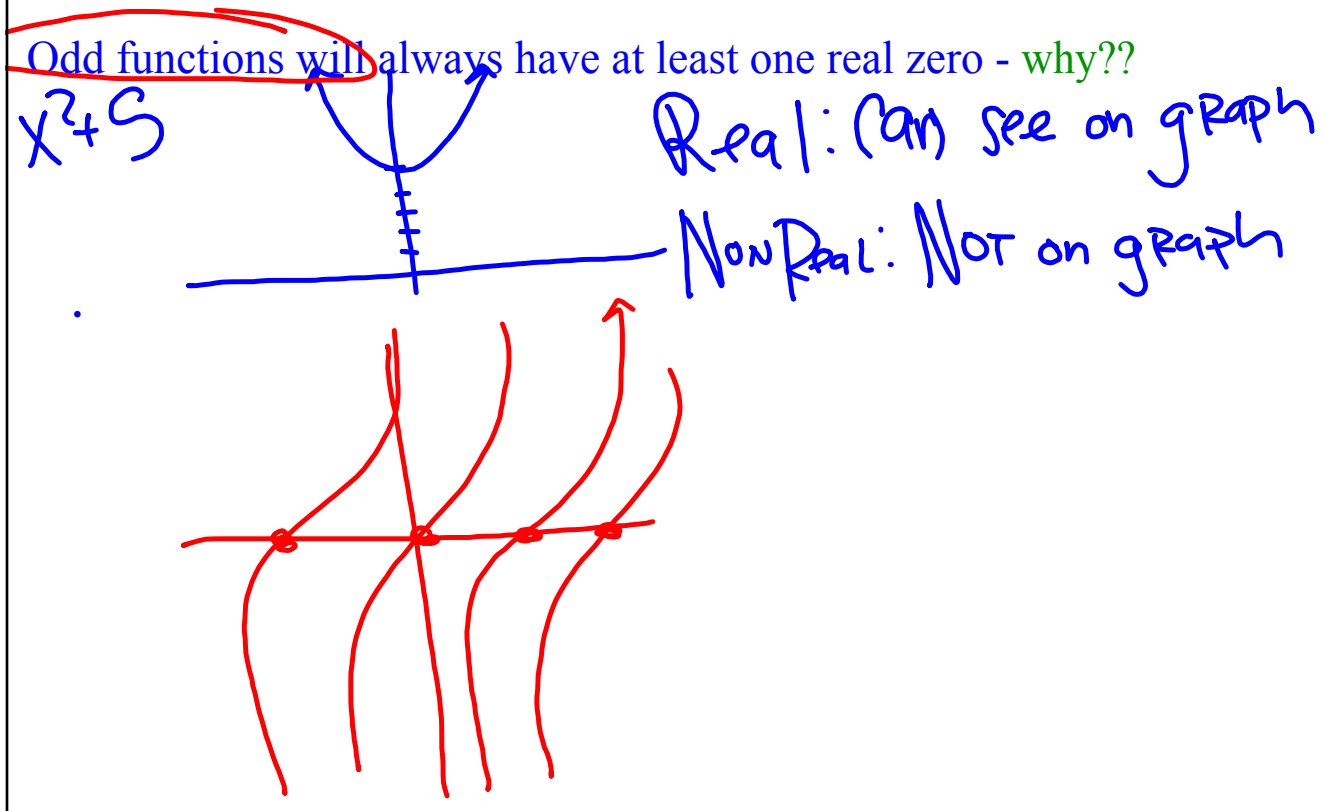
Complex Conjugates: complex imaginary factors come in conjugate pairs

conjugate $\rightarrow a-bi$

(if 3i is a zero, - 3i is also)

Odd functions will always have at least one real zero - why??

x^3+5



Find all zeros of $p(x) = x^3 - 125$. Include any multiplicities greater than 1.

First factor the difference of two cubes.

Find all zeros of $p(x) = x^4 - 256$. Include multiplicities greater than 1.

Find use factoring patterns to factor the polynomial.

$x^4 - 256$ $\quad a = x^2 \quad b = 4$

$a = x^2$
$b = 16$

$(x^2+16)(x^2-16)$

$(x^2+16)(x+4)(x-4)$

$x^2 + 16$

$a = 1$
$b = 0$
$c = 16$

$x = \dfrac{0 \pm \sqrt{0 - 4(1)(16)}}{2}$

$x = \dfrac{\pm\sqrt{-64}}{2} = \dfrac{\pm 8i}{2} = \pm 4i$

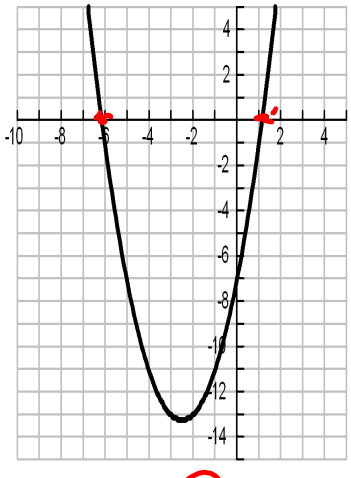
$(x+4i)(x-4i)(x+4)(x-4)$

ZEROS: $-4, 4, 4i, -4i$

How many complex zeros does each function have? How many are real? How many are non-real?

total

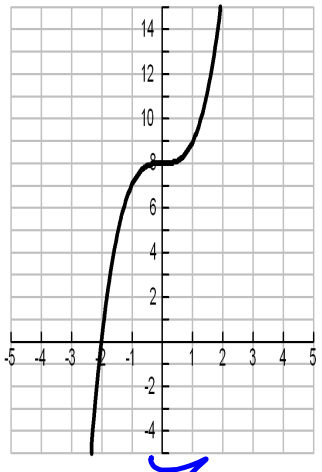
$x^2 + 5x - 7$



C : 2

R : 2 NR : 0

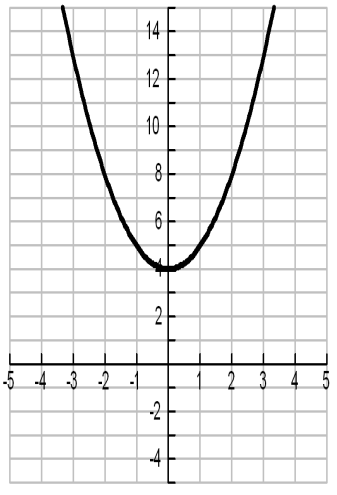
$x^3 + 8$



C : 3

R : 1 NR : 2

$x^2 + 4$



C : 2

R : 0 NR : 2

Linear Factorization Thm: a polynomial of nth degree has n linear factors

(some factors may be complex imaginary)

$$x^4 - 6x^3 + 10x^2 - 6x + 9$$

$$(x-3)(x+3)(x-i)(x+i)$$

ZEROS: $x = -3, 3, -i, i$

Find all zeros and write a linear factorization of the following polynomial:

x^3+5x^2+x+5

C: 3

R: 1 NR: 2

$X = -5, i, -i$ ← 3 ZEROS

$-5 \mid 1 \quad 5 \quad 1 \quad 5$

$\quad\quad\downarrow \quad -5 \quad 0 \quad -5$

$\quad\quad 1x^2 + 0x + 1 \quad | \quad 0$

$a \quad b \quad c$

$(x+5)(x+i)(x-i)$

LINEAR FACTORIZATION

$x = \frac{0 \pm \sqrt{0-4(1)(1)}}{2(1)}$

$X = \frac{\pm\sqrt{-4}}{2} = \frac{\pm 2i}{2} = \pm i$

$x^3-11x^2+49x-75$

$X = 3, 4+3i, 4-3i$ 3 ZEROS

C: 3

R: 1 NR: 2

$3 \mid 1 \quad -11 \quad 49 \quad -75$

$\quad\quad\downarrow \quad 3 \quad -24 \quad 75$

$\quad\quad 1x^2 - 8x + 25 \quad | \quad 0$

$X = \frac{8 \pm \sqrt{64 - 4(1)(75)}}{2} = \frac{8 \pm \sqrt{-36}}{2}$

$(x-3)(x-4+3i)(x-4-3i)$

LINEAR FACTORIZATION

$= \frac{8 \pm 6i}{2} = 4 \pm 3i$

Find all zeros and write a linear factorization of the following polynomial:

$x^4 + x^3 + 5x^2 - x - 6$ $X = -1, 1$

C : 4

R : 2 NR : 2

-1	1	1	5	-1	-6
	↓	-1	0	-5	6
	1	0	5	-6	0
1	1	0	5	-6	
	↓	1	1	6	
	$1x^2$	+ $1x$	+ 6	0	

$X = \frac{-1 \pm \sqrt{1 - 4(1)(6)}}{2} = \frac{-1 \pm \sqrt{-23}}{2} : \frac{-1 \pm \sqrt{23}\,i}{2}$

$X = -1, 1, \frac{-1 + \sqrt{23}\,i}{2}, \frac{-1 - \sqrt{23}\,i}{2}$

$(x+1)(x-1)\left(x + \frac{1 + \sqrt{23}\,i}{2}\right)\left(x + \frac{1 - \sqrt{23}\,i}{2}\right)$

Use the given zero to find the remaining zeros and write a linear factorization:

$2i$; $x^4+10x^3+38x^2+40x+136$

C: 4

R: 0 NR: 4

$x = 2i, -2i$

$2i$ | 1 10 38 40 136

$2i(10+2i)$

$20i + 4i^2(-1) = -4+20i$

$2i(34+20i)$

$68i + 40i^2 = -136i^2(-1)$

	1	10	38	40	136
$2i$		$2i$	$-4+20i$	$-40+68i$	-136
	1	$10+2i$	$34+20i$	$68i$	0
$-2i$		$-2i$	$-20i$	$-68i$	
	$1x^3$	$+10x$	$+34$		0

$$x = \frac{-10 \pm \sqrt{10 - 4(1)(34)}}{2} = \frac{-10 \pm \sqrt{-36}}{2}$$

$$= \frac{-10 \pm 6i}{2}$$

$x = 2i, -2i, -5+3i, -5-3i$

$(x+2i)(x-2i)(x+5+3i)(x+5-3i) = -5 \pm 3i$

Write a polynomial function of minimum degree with the following zeros and multiplicities:

4, 7, 2i, -2i

$(x-4)(x-7)(x+2i)(x-2i)$

- 4, 2 + 3i, 2 - 3i

$(x+4)(x-2+3i)(x-2-3i)$

3 with multi of 2

5 + i with multi of 1

$(x-3)^2(x-5+i)(x-5-i)$