3.1 Zeros of a Polynomial

Objectives:

- I can find the zeroes of a polynomial by using the factor theorem, remainder theorem, and rational roots theorem
- I know the difference between a zero & a factor

Divide the following polynomials

$x+4\overline{)3x^2+7x-20}$

$x+4=0 \quad x=-4$

$$\begin{array}{r|rrr} -4 & 3 & 7 & -20 \\ & \downarrow & -12 & 20 \\ \hline & 3x & -5 & \boxed{0} \end{array}$$

Factors: $(x+4)(3x-5)$

Zeros: $x=-4, 5/3$

$3x-5=0$
$+5 \quad +5$

$\frac{3x}{3}=\frac{5}{3} \quad x=5/3$

$$\frac{2x^4-5x^3+7x^2-3x+1}{x-3}$$

$$\begin{array}{r|rrrrr} 3 & 2 & -5 & 7 & -3 & 1 \\ & \downarrow & 6 & 3 & 30 & 81 \\ \hline & 2x^3 & 1x^2 & 10x & 27 & \boxed{82} \end{array}$$

Not a factor

Identify the zeros of the following and explain what that means graphically.

factors

factor form → $f(x)=(x+2)(x-1)(x+3)$ looks like $(x \pm \#)$

FACTOR: DIVIDES EVENLY INTO POLY.

ZERO: WHAT MAKES a factor $=0$ looks like $x=\#$

zeros: $x=-2, 1, -3$ WHERE I CROSS x axis

Write the function in standard form and state the relationship between the degree and zeros of the function

SF: $f(x)=\underbrace{(x+2)(x-1)}(x+3)$

$x^2-1x+2x-2$

$(x^2+x-2)(x+3)$

$f(x)=x^3+3x^2+x^2+3x-2x-6$

SF: $f(x)=x^3+4x^2+x-6$

Degree = # of zeros

~~Remainder Theorem:~~

For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$

$p(4)$

4| . . . .

$\boxed{p(4)}$

*

Factor Theorem:

If the remainder in $p(x) = (x - a)q(x) + p(a)$ is 0, then $p(x) = (x - a)q(x)$, which tells you that $(x - a)$ is a factor of $p(x)$.

Conversely, if $(x - a)$ is a factor of $p(x)$, then you can write $p(x)$ as $p(x) = (x - a)q(x)$, and when you divide $p(x)$ by $(x - a)$, you get the quotient $q(x)$ with a remainder of 0.

Remainder $= 0 \longrightarrow$ Is a factor

Remainder $\neq 0 \longrightarrow$ Not a factor

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Determine whether the given binomial is a factor of the polynomial p(x). If so, find the remaining factors of p(x).

(B) $p(x) = x^4 - 4x^3 - 6x^2 + 4x + 5;$ $\boxed{(x + 1)}$

$$
\begin{array}{r|rrrrr}
-1 & 1 & -4 & -6 & +4 & +5 \\
 & \downarrow & -1 & 5 & 1 & -5 \\
\hline
 & 1 & -5 & -1 & 5 & \boxed{0}
\end{array}
$$

yes, $(x+1)$ is a factor

~~So, $p(x) = x^4 - 4x^3 - 6x^2 + 4x + 5 =$ __________.~~

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Example 3 Determine whether the given binomial is a factor of the polynomial $p(x)$. If so, find the remaining factors of $p(x)$.

(A) $p(x) = x^3 + 3x^2 - 4x - 12; (x + 3)$

$$\begin{array}{r|rrrr} -3 & 1 & 3 & -4 & -12 \\ & \downarrow & -3 & 0 & 12 \\ \hline & 1x^2 & 0x & -4 & \boxed{0} \end{array}$$

yes, $(x+3)$ is a factor

$f(x) = (x+3)(x^2-4)$

$f(x) = (x+3)(x+2)(x-2)$

zeros $x = -3, -2, 2$

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Your Turn

Determine whether the given binomial is a factor of the polynomial $p(x)$. If it is, find the remaining factors of $p(x)$.

8. $p(x) = 2x^4 + 8x^3 + 2x + 8; (x + 4)$

$$\begin{array}{r|rrrrr} -4 & 2 & 8 & 0 & 2 & 8 \\ & \downarrow & -8 & 0 & 0 & -8 \\ \hline & 2x^3 & 0x^2 & 0x & 2 & \boxed{0} \end{array}$$

$(x+4)(2x^3+2)$

$2(x+4)(x^3+1)$

$2(x+4)(x+1)(x^2+x+1)$

9. $p(x) = 3x^3 - 2x + 5; (x - 1)$

$$\begin{array}{r|rrrr} 1 & 3 & 0 & -2 & 5 \\ & \downarrow & 3 & 3 & 1 \\ \hline & 3 & 3 & 1 & \boxed{6} \end{array}$$

NOT a factor

Rational Root Theorem:

If all coefficients are integers and the constant is not 0, then all possible rational roots are:

$$x = \pm \frac{\text{factors of constant}}{\text{factors of } \underbrace{\text{leading coefficient}}_{\text{1st coefficient in standard form}}}$$

Find the rational zeros of the polynomial function; then write the function as a product of factors.

$$f(x) = x^3 + 2x^2 - 19x - 20$$

constant

LCE

Factors of Constant: $\dfrac{\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20}{\pm 1}$

Possible: $\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$

Find the rational zeros of the polynomial function; then write the function as a product of factors.

$$f(x) = x^4 - 4x^3 - 7x^2 + 22x + 24$$

Possible: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$

$$\begin{array}{r|rrrr|r} -1 & 1 & -4 & -7 & 22 & 24 \\ & \downarrow & -1 & 5 & 2 & -24 \\ \hline & 1x^3 & -5x^2 & -2x & 24 & 0 \end{array}$$

$$\begin{array}{r|rrr|r} -2 & 1x^3 & -5x^2 & -2x & 24 \\ & \downarrow & -2 & 14 & -24 \\ \hline & 1x^2 & -7x & +12 & 0 \end{array}$$

$(x-3)(x-4)$

zeros:

$x = -1, -2, 3, 4$

factors:

$(x-3)(x-4)(x+1)(x+2)$

Find all the zeros $f(x) = x^3 - 2x^2 - 8x$

$x(x^2-2x-8)$

$x(x-4)(x+2)$

$$\begin{array}{c|c} \multicolumn{2}{c}{-8} \\ \hline -4 & 2 \end{array}$$

zeros: $x = 0, 4, -2$

$x = 0$

$0 = 0$

Find all the zeros of: $2x^4 - 7x^3 - 8x^2 + 14x + 8$

$$
\begin{array}{r|rrrrr}
4 & 2 & -7 & -8 & 14 & 8 \\
 & \downarrow & 8 & 4 & -16 & -8 \\
\hline
 & 2x^3 & 1x^2 & -4x & -2 & \boxed{0}
\end{array}
$$

Find all the zeros of: $f(x) = x^3 + x^2 - 14x + 6$

$$\begin{array}{r|rrrr} 3 & 1 & 1 & -14 & 6 \\ & \downarrow & 3 & 12 & -6 \\ \hline & 1x^2 & 4x & -2 & \boxed{0} \end{array}$$

$a = 1$
$b = 4$
$c = -2$

$(x^2 + 4x - 2)$

$$\begin{array}{c|c} & -2 \\ \hline -2 + & 1 = -1 \\ 2 + & -1 = 1 \end{array}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{16 - 4(1)(-2)}}{2(1)}$$

$$= \frac{-4 \pm \sqrt{16 + 8}}{2} = \frac{-4 \pm \sqrt{24}}{2} = -2 \pm \frac{\sqrt{24}}{2}$$

$$\boxed{\text{zeros: } x = 3,\ -2 + \frac{\sqrt{24}}{2},\ -2 - \frac{\sqrt{24}}{2}}$$

Find the polynomial function with a leading coefficient of 2 that has the given degree and zeros: degree 3, zeros -2, 4, 1

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