3-1 Arithmetic and Geometric Growth

add

multiply

Objectives:

I can determine the difference between an arithmetic and geometric sequence.

I can find a common difference or common factor from a sequence.

I can write the explicit and recursive equation for a sequence.

Vocabulary

Rate of change: what a sequence changes by

First Term: f
1st term in sequence a_1

Arithmetic: adds/subtract

Common difference: d
what I add or subtract each time

Geometric: multiply

Common Ratio: r
what I multiply by each time

Recursive Function: adds on to itself, using previus term

Recursive Function: uses previous term to multiply

Explicit Function: uses term # & rate of change

Explicit Function:

Recursive: $a_n = a_{n-1} \pm d, a_1 = f$

Explicit $a_n = d(n-1) \pm f$

Recursive: $a_n = a_{n-1} \cdot r, a_1 = f$

Explicit $a_n = f \cdot r^{n-1}$

Notation

a_n: TERM TRYING TO FIND

n: TERM #

a_{n-1}: PREVIOUS TERM

a_{n+1}: NEXT TERM

$\ldots a_{n-1}, a_n, a_{n+1}, a_{n+2} \ldots$

+5 +5

3 , 8 , 13 , 18 , 23,...

Next 3 terms: 28, 33, 38

Arithmetic or Geometric?

Common Difference/Ratio:

$d = 5 \quad f = 3$

Recursive:

$a_n = a_{n-1} + 5, a_1 = 3$

Explicit: $a_n = 5(n-1) + 3$

$a_{100} = 5(100-1) + 3$

$= 5(99) + 3$

$=$

-2 -2 -2

11, 9, 7, 5, ...

Next 3 terms: 3, 1, -1

Arithmetic or Geometric?

Common Difference/Ratio:

$d = -2$ $f = 11$

Recursive: $a_n = a_{n-1} - 2;\ a_1 = 11$

Explicit!

$a_n = -2(n-1) + 11$

$\times 2$ $\times 2$

2, 4, 8, 16,...

Next 3 terms: 32, 64, 128

Arithmetic or Geometric?

Common Difference/Ratio:

$r = 2$ $f = 2$

R: $a_n = a_{n-1} \cdot 2$ $a_1 = 2$

E: $a_n = 2 \cdot 2^{n-1}$

$a_5 = 2 \cdot 2^{5-1}$

$= 2 \cdot 2^4$

$= 32$

3 , 9 , 27, ...

Next 3 terms: 81, 243, 729

Arithmetic or Geometric?

Common Difference/Ratio: $r = 3$, $f = 3$, $a_1 = 3$

R: $a_n = a_{n-1} \cdot 3$

E: $a_n = 3 \cdot 3^{n-1}$

In the following figure, each triangle is created by 3 toothpicks

Figure 1 Figure 2 Figure 3

$d = 2$

$f = 3$

Figure	Toothpicks
1	3
2	5
3	7
4	9
5	11

Explicit:

$a_n = 2(n-1) + 3$

Recursive:

$a_n = a_{n-1} + 2 \; ; \; a_1 = 3$

You've saved up over $100 to buy you new iPhone. You borrow $470 from your parents to pay the rest. If you pay them back $20 a week, ~~starting with the 2nd week~~, how many weeks will it be until you owe them $350?

470, 450, 430

1 2 3

$d = -20$

$f = 470$

$a_n = -20(n-1) + 470$

$350 = -20(n-1) + 470$

$350 = -20n + 20 + 470$

$350 = -20n + 490$

$-490 \quad -490$

$-140 = -20n$

$\frac{-140}{-20} = \frac{-20n}{-20}$

$7 = n$

7 weeks