

3-1 Arithmetic and Geometric Growth

↑
add

↑
multiply

Objectives:

I can determine the difference between an arithmetic and geometric sequence.

I can find a common difference or common factor from a sequence.

I can write the explicit and recursive equation for a sequence.

Vocabulary

Rate of change:
What a sequence changes by

Arithmetic:

adds/subtract

Common difference:

What I add or subtract each time

Recursive Function:

adds on to itself, using previous

Explicit Function:

USES TERM #
Rate of change

Recursive:

$$a_n = a_{n-1} \pm d, a_1 = f$$

EXPLICIT

$$a_n = d(n-1) \pm f$$

First Term:

1st term in sequence a_1

Geometric:

Multiply

Common Ratio:

WHAT I multiply by each time

Recursive Function:

USES PREVIOUS TERM TO MULTIPLY

Explicit Function:

Recursive:

$$a_n = a_{n-1} \cdot r, a_1 = f$$

EXPLICIT

$$a_n = f \cdot r^{n-1}$$

Notation

a_n : TERM TRYING
TO FIND

n : TERM #

a_{n-1} : Previous
TERM

a_{n+1} : Next
TERM

... .. $a_{n-1}, a_n, a_{n+1}, a_{n+2}, \dots$

$$+5 +5$$

3, 8, 13, 18, 23, ...

Next 3 terms: 28, 33, 38

Arithmetic or Geometric?



Common Difference/Ratio:

$$d = 5 \quad f = 3$$

Recursive:

$$a_n = a_{n-1} + 5, a_1 = 3$$

$$\text{Explicit: } a_n = 5(n-1) + 3$$

$$\begin{aligned} a_{100} &= 5(100-1) + 3 \\ &= 5(99) + 3 \\ &= \end{aligned}$$

^{-2 -2 -2}
11, 9, 7, 5, ...

Next 3 terms: 3, 1, -1

Arithmetic or Geometric?

Common Difference/Ratio:

$$d = -2 \quad f = 11$$

Recursive: $a_n = a_{n-1} - 2; a_1 = 11$

Explicit:

$$a_n = -2(n-1) + 11$$

$\times 2 \checkmark$
 2, 4, 8, 16, ...

Next 3 terms:

32, 64, 128

Arithmetic or Geometric?

Common Difference/Ratio:

$$r=2 \quad f=2$$

$$R: a_n = a_{n-1} \cdot 2 \quad a_1 = 2$$

$$E: a_n = 2 \cdot 2^{n-1}$$

$$\begin{aligned} a_5 &= 2 \cdot 2^{5-1} \\ &= 2 \cdot 2^4 \\ &= 32 \end{aligned}$$

3, 9, 27, ...

Next 3 terms: 81, 243, 729

Arithmetic or Geometric?

Common Difference/Ratio: $r = 3$

R: $a_n = a_{n-1} \cdot 3$

$f = 3$
 $a_1 = 3$

E: $a_n = 3 \cdot 3^{n-1}$

In the following figure, each triangle is created by 3 toothpicks



Figure 1



Figure 2

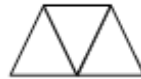


Figure 3

Figure	Toothpicks
1	3
2	5
3	7
4	9
5	11

$$d = 2$$

$$f = 3$$

Explicit:

$$a_n = 2(n-1) + 3$$

Recursive:

$$a_n = a_{n-1} + 2 ; a_1 = 3$$

You've saved up over \$100 to buy your new iPhone. You borrow \$470 from your parents to pay the rest. If you pay them back \$20 a week, starting with the 2nd week, how many weeks will it be until you owe them \$350?

$$470, 450, 430$$

1 2 3

$$d = -20$$

$$f = 470$$

$$a_n = -20(n-1) + 470$$

$$\downarrow$$

$$350 = -20(n-1) + 470$$

$$350 = -20n + 20 + 470$$

$$350 = -20n + 490$$

$$\begin{array}{r} -490 \\ \hline \end{array}$$

$$-140 = -20n$$

$$\begin{array}{r} -70 \\ \hline \end{array}$$

$$7 = n$$

7 weeks

