1-3 Piecewise and Step Functions

Objectives:

-I can graph a piecewise function

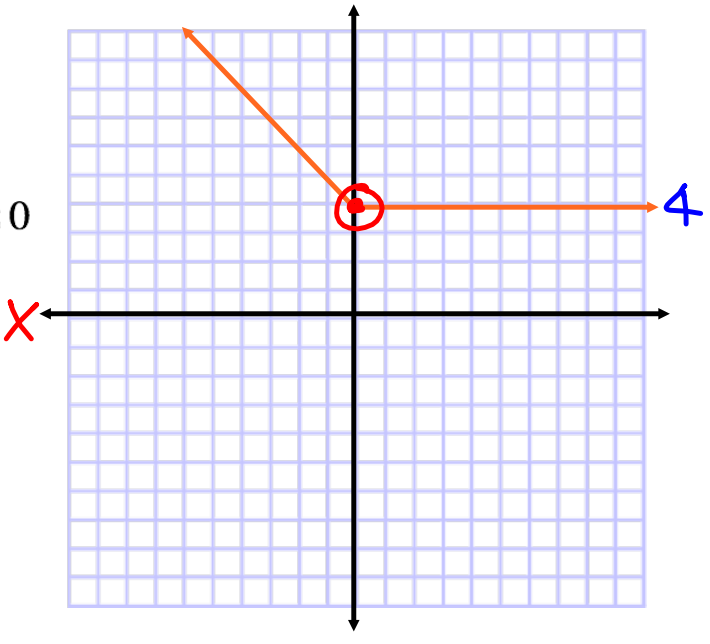
- I can write the equation of a piecewise function

-I can identify least and greatest integer functions

A piecewise function is a function with a different equations defined over unique intervals of x.

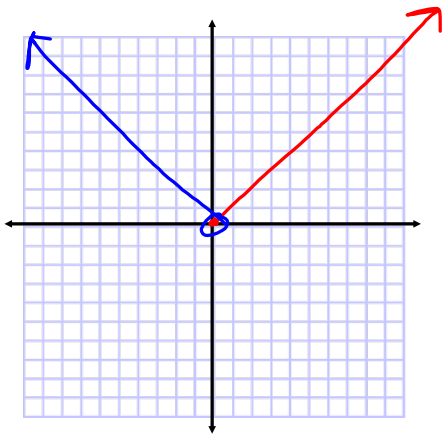
For example:

$$f(x)=\begin{cases}-x+4 \ \ if \ \ x\le 0\\ 4 \ \ if \ \ x>0\end{cases}$$

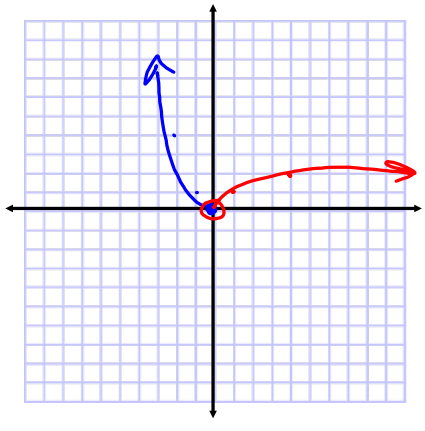


Graph the following:

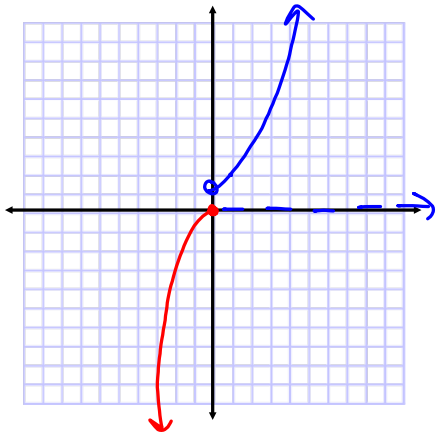
$$f(x)=\begin{cases} x & \text{if } x\geq 0 \\ -x & \text{if } x<0 \end{cases}$$



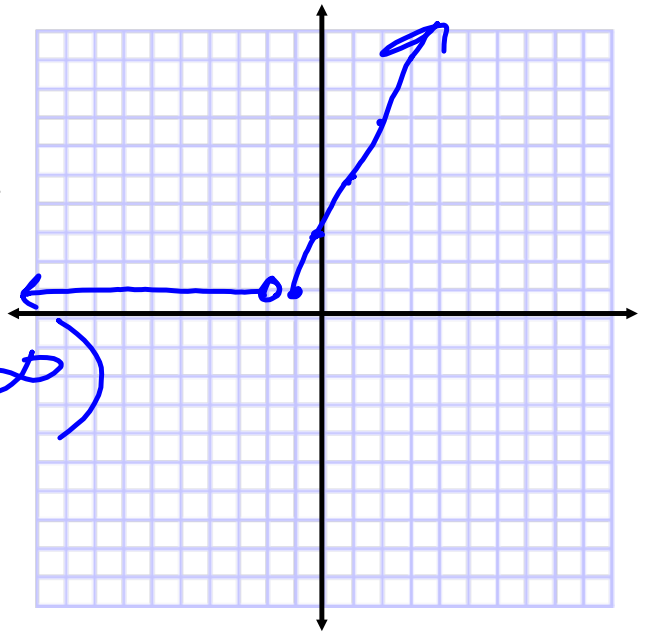
$$f(x)=\begin{cases} x^2 & \text{if } x\leq 0 \\ \sqrt{x} & \text{if } x>0 \end{cases}$$



$$f(x)=\begin{cases} x^3 & \text{if } x\leq 0 \\ 2^x & \text{if } x>0 \end{cases}$$



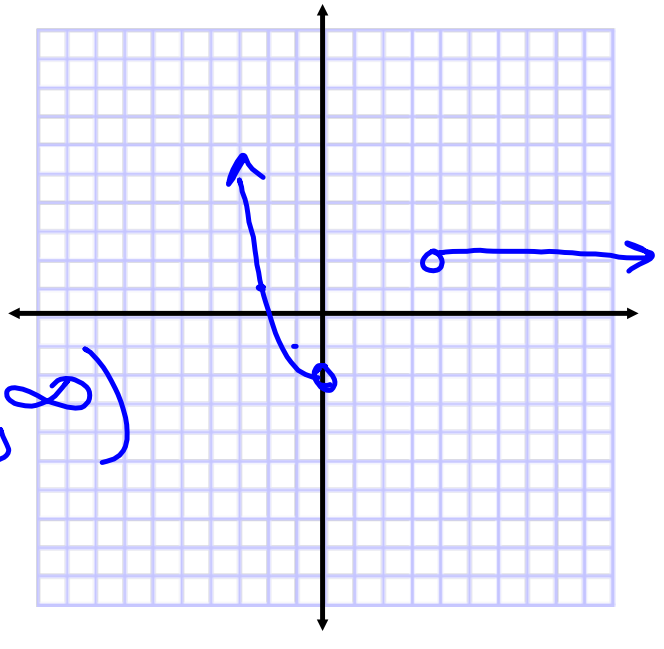
$$f(x)=\begin{cases}1, & x<-2\\ 2x+3, & x\geq -1\end{cases}$$



Domain

$(-\infty,-2)\cup[-1,\infty)$

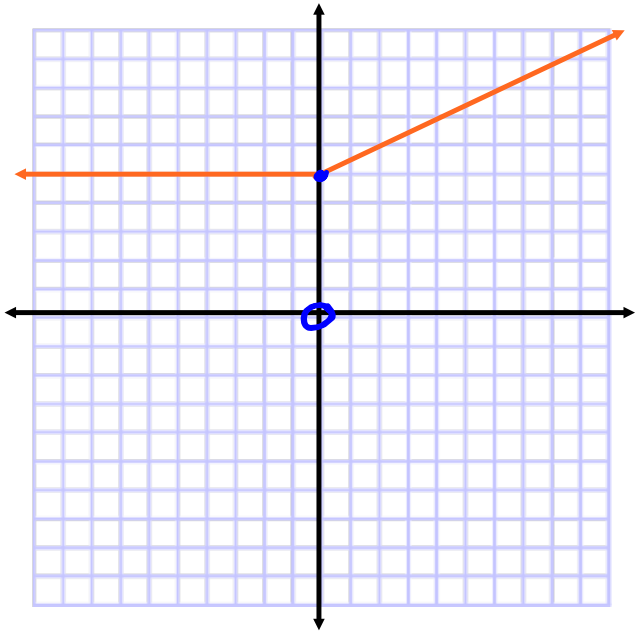
$$f(x)=\begin{cases}x^2-2, & x<0\\ \sqrt{x}, & x>4\end{cases}$$



Domain

$(-\infty,0)\cup(4,\infty)$

Write the equation for the following piecewise functions



$$f(x)=\begin{cases}5, & x\le 0\\ \frac{1}{2}x+5, & x>0\end{cases}$$

Problem 2: Taxi Fares

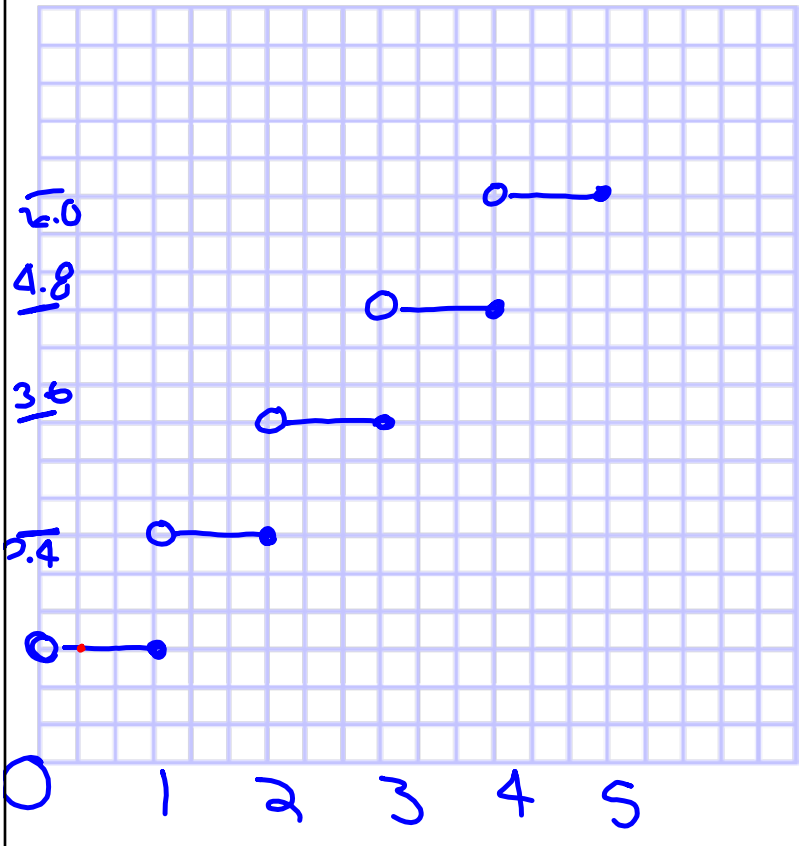
In 2006, the rate for a taxi ride in Macon, Georgia, was $1.20 for the first mile or part of a mile, and $1.20 for each additional mile or part of a mile.

1. Define a piecewise function, g(x), for the cost of a taxi ride up to 5 miles.

$$g(x) = \begin{cases} 1.20, & 0 < x \leq 1 \\ 2.40, & 1 < x \leq 2 \\ 3.60, & 2 < x \leq 3 \\ 4.80, & 3 < x \leq 4 \\ 6.00, & 4 \leq x \leq 5 \end{cases}$$

2. What is the slope of each interval? Explain your reasoning.

3. Graph g(x) for x < 5 miles.



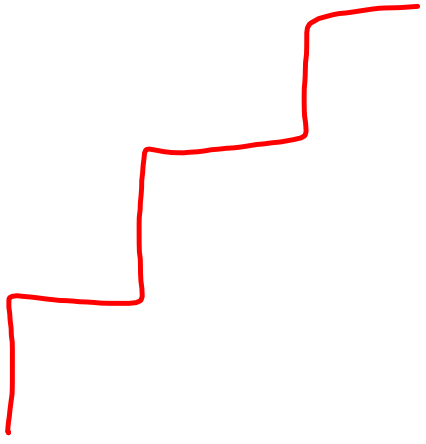
$$f(x) = \begin{cases} 1.2 \\ 2.4 \\ 3.6 \\ 4.8 \\ 6.0 \end{cases}$$

4. Describe the graph of the function as either increasing or decreasing.

Increasing

You have just graphed a *step function. A* ***step function*** is a piecewise function whose pieces are disjoint constant functions.

5. Why do you think this function is called a step function?



Problem 3 Special Step Functions

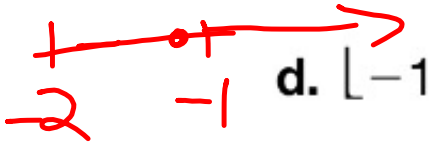
The *greatest integer function* is a special kind of step function. The **greatest integer function**, also known as the **floor function**, $G(x) = \lfloor x \rfloor$ is defined as the greatest integer less than or equal to x.

1. Evaluate each using the greatest integer function.

a. $\lfloor 2 \rfloor = 2$

b. $\lfloor 0.17 \rfloor = 0$

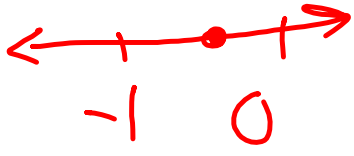
c. $\lfloor 2.34 \rfloor = 2$



d. $\lfloor -1.2 \rfloor = -2$

e. $\lfloor 2.99999 \rfloor = 2$

f. $\lfloor -0.2 \rfloor = -1$



The *least integer function* is another special kind of step function.
The **least integer function** $L(x) = \lceil x \rceil$ also known as the **ceiling function,** is defined as the least integer greater than or equal to x.

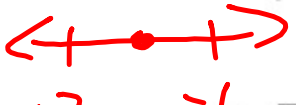
3. Calculate each:

a. $\lceil 2 \rceil = 2$

b. $\lceil 0.17 \rceil = 1$

c. $\lceil 2.34 \rceil = 3$

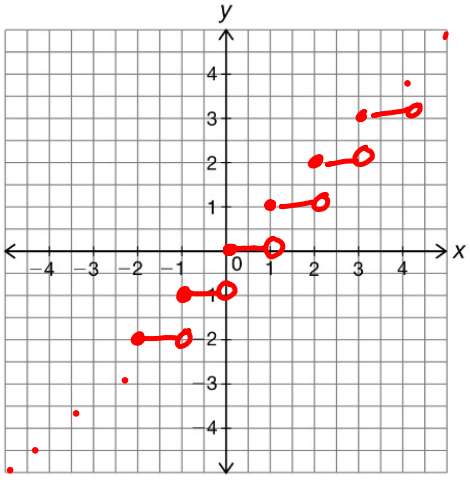
d. $\lceil -1.2 \rceil = -1$



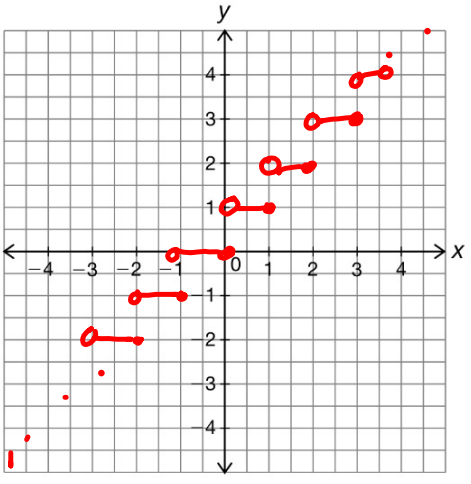
e. $\lceil 2.99999 \rceil = 3$

f. $\lceil -0.2 \rceil = 0$

Graph $G(x) = \lfloor x \rfloor$.



Graph $L(x) = \lceil x \rceil$.



$\lfloor 1 \rfloor = 1$

$\lceil .1 \rceil = 1$

$\lfloor x \rfloor \quad x < 0$